Instructions: This homework assignment covers **Chapter 10** of the course notes. On each part, there is opportunity for partial credit, so show all of your work and explain all of your reasoning. No work/no explanation means no credit even if your answer is correct. If you use R to answer any part or to check your work, please include all code and output as attachments. Do not just write out the code you used.

1. A rocket motor is manufactured by bonding together two types of propellants, an igniter and a sustainer. The shear strength of the bond Y (measured in psi) is thought to be a linear function of the age of the propellants x (measured in weeks) when the motor is cast. Investigators observed a sample of $n = 20$ motors and recorded the bond shear strength and propellent age on each motor. Here are the data:

Motor	Strength (Y)	Age (x)
1	2158.70	15.50
2	1678.15	23.75
3	2316.00	8.00
4	2061.30	17.00
5	2207.50	5.00
6	1708.30	19.00
7	1784.70	24.00
8	2575.00	2.50
9	2357.90	7.50
10	2277.70	11.00
11	2165.20	13.00
12	2399.55	3.75
13	1779.80	25.00
14	2336.75	9.75
15	1765.30	22.00
16	2053.50	18.00
17	2414.40	6.00
18	2200.50	12.50
19	2654.20	2.00
20	1753.70	21.50

(a) Prepare a scatterplot for these data. Does a simple linear regression model

$$Y = \beta_0 + \beta_1 x + \epsilon$$

seem reasonable for the population of motors?
(b) Estimate the population-level model in part (a) using least squares and superimpose the estimate over your scatterplot in part (a).
(c) Calculate the 20 fitted values and 20 residuals from your least-squares fit in part (b). Verify, up to rounding error, the residuals sum to 0.
(d) Write a 95% confidence interval for β_1 and interpret what it means.

2. In the manufacture of commercial wood products, it is important to estimate the relationship between the stiffness of a wood product (Y, measured in lbs/in^2) and its density (x, measured in lbs/ft^3). Thirty industrial particle boards were produced at different densities and the stiffness of each board was measured.

Here are the data:

Board	Stiffness (Y)	Density (x)	Board	Stiffness (Y)	Density (x)
1	14814	9.5	16	17502	8.4
2	14007	9.8	17	19442	11.0
3	7573	8.3	18	14191	9.9
4	9714	8.6	19	8076	6.4
5	5304	7.0	20	10728	8.2
6	43243	17.4	21	25319	15.0
7	28028	15.2	22	41792	16.4
8	49499	16.7	23	25312	15.4
9	26222	15.0	24	22148	14.5
10	26751	14.8	25	18036	13.6
11	96305	25.6	26	104170	23.4
12	72594	24.4	27	49512	23.3
13	32207	19.5	28	48218	21.2
14	70453	22.8	29	47661	21.7
15	38138	19.8	30	53045	21.3

(a) Prepare a scatterplot for these data. Does a simple linear regression model

$$Y = \beta_0 + \beta_1 x + \epsilon$$

seem reasonable for the population of particle boards?
(b) Estimate the population-level model in part (a) using least squares and superimpose the estimate over your scatterplot in part (a). Do you have any concerns about your model fit?
(c) Use your model fit in part (b) to write a

- 95% confidence interval for the population mean stiffness $E(Y)$ for particle boards whose density is $x_0 = 15.0$ lbs/ft^3
- 95% prediction interval for a single particle board whose density is $x_0 = 15.0$ lbs/ft^3.

Interpret each interval.
(d) What value of the density x will produce confidence intervals for $E(Y)$ and prediction intervals for Y^* with minimum length? Explain why this makes sense. Give a value of the density x for which inference for $E(Y)$ and Y^* would be unreasonable or even nonsensical. Explain.