GROUND RULES:

- Print your name at the top of this page. Do not put your name on any other page.
- This is a closed-book and closed-notes exam. A list of discrete and continuous distributions appears at the end.
- This exam contains 7 problems, each worth 10 points. This exam is worth **70 points** total.
- You may use a calculator, but this calculator cannot have internet access. You cannot use your phone as a calculator. You cannot share calculators with another student. Show all of your work; use a calculator only to do final calculations or to check your work.
- Each problem contains parts. On each part, there is opportunity for partial credit, so show all of your work and explain all of your reasoning. **Translation:** No work/no explanation means no credit.
- On any problem, you may use the back of the page if you need more space. I also have extra paper if you need it.
- Any discussion or inappropriate communication between you and another examinee, as well as the appearance of any unnecessary material, will result in a declaration of academic dishonesty. Don't risk it!
- You have 75 minutes to complete this exam.

HONOR PLEDGE FOR THIS EXAM:

After you have finished the exam, please read the following statement and sign your name below it.

> *I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own.*

1. You need an itinerary to visit 6 cities in succession. The 6 cities are

Boise, ID	Seattle, WA	Denver, CO
Miami, FL	Atlanta, GA	Newark, NJ

For example, one itinerary is

$$\text{Atlanta} \longrightarrow \text{Denver} \longrightarrow \text{Newark} \longrightarrow \text{Boise} \longrightarrow \text{Miami} \longrightarrow \text{Seattle.}$$

Note that the ordering of the cities uniquely determines the itinerary. All 6 cities must be included.

(a) How many possible itineraries are there?

(b) What does it mean when we say "each possible itinerary is equally likely?"

(c) Suppose you pick one itinerary at random from all possible itineraries. What is the probability the itinerary you select has the West coast cities (Boise, Seattle, and Denver) and the East cost cities (Miami, Atlanta, and Newark) grouped together? *Hint:* This means you visit the 3 West coast cities first and then the 3 East coast cities second (or vice versa).

2. A construction company hires two engineers to bid on potential jobs: Engineer 1 and Engineer 2.

- Engineer 1 bids on 70% of the jobs; Engineer 2 bids on the remaining jobs.
- Among all jobs on which Engineer 1 bids, an error is made on 2% of the bids.
- Among all jobs on which Engineer 2 bids, an error is made on 4% of the bids.

(a) Define two relevant events A and B using the information above. Interpret each percent above as a probability (two are conditional probabilities). Do not define more than two events or you will be making the problem too hard.

(b) What proportion of all bids will contain an error? Which "law" are you using here?

(c) Suppose a new bid sent to the manager contains an error. Find the probability this bid was prepared by Engineer 1.

3. At a local plant, the random variable

$$X = \text{number of serious accidents per year}$$

has the following probability mass function (pmf):

x	0	1	2	3	4
$p_X(x)$	0.5	0.2	0.1	0.1	0.1

(a) Explain why X is a discrete random variable (and not continuous).

(b) Find the mean number of serious accidents per year.

(c) Prepare a graph of $F_X(x)$, the cumulative distribution function of X. Label both axes and use appropriate tick marks on each axis. Neatness counts.

4. A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates the defective rate of the device is 3%.

(a) An inspector randomly picks 10 devices and tests them. What is the probability there are 1 or 2 defective devices? *Hint:* Use the binomial distribution.

(b) Your calculation in part (a) requires three assumptions. Describe what these assumptions are in this problem.

5. A gas station is supplied with gasoline once per day. The station's daily sales (in 10,000s of gallons) is a continuous random variable X with probability density function (pdf)

$$f_X(x) = \begin{cases} 0.25(2-x)^3, & 0 < x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $P(X < 0.4)$, the probability daily sales are less than 4000 gallons.

(b) The station owner wants to determine the supply c so that the probability the daily sales X exceeds c is 0.01. Determine what c is. *Hint:* What quantile (percentile) of X does the owner want to determine?

6. The number of automobiles entering a remote mountain tunnel has a Poisson distribution with mean $\lambda = 10$ cars per hour.

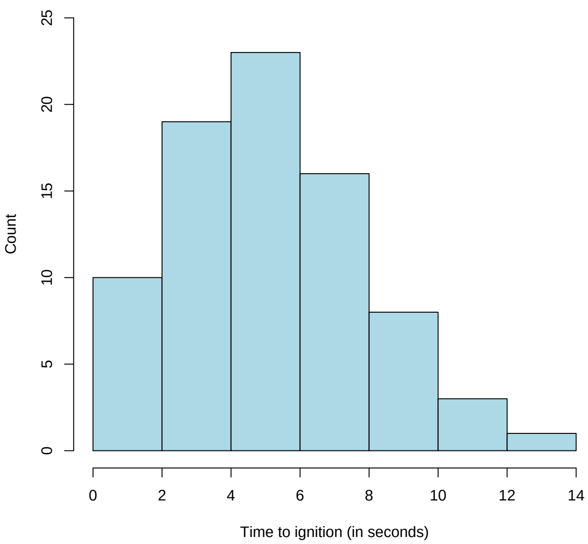
(a) Find the probability there are at most 2 cars that enter the tunnel in a given hour.

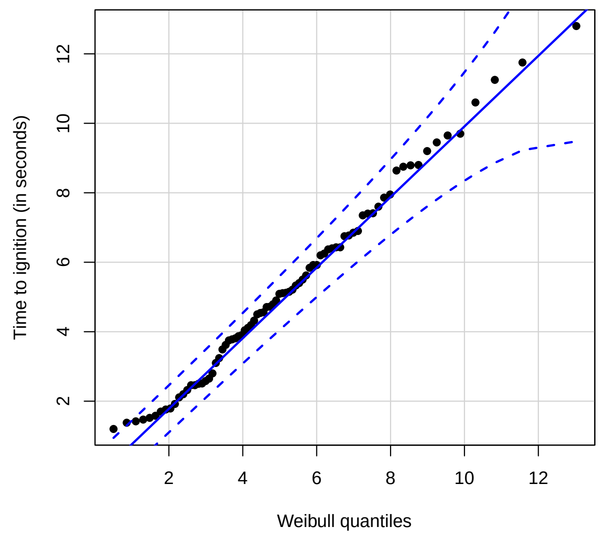
(b) In any given hour, what is the probability we would have to wait at least 30 minutes for the **first** car to enter the tunnel? Note that 30 minutes is 1/2 of an hour.

7. In a fire safety study, engineers collected a random sample of $n = 80$ upholstery pieces and on each piece recorded T, the ignition time (in seconds) when exposed to a flame. Here are the data:

2.58	2.51	4.04	6.43	1.58	4.32	2.20	4.19	4.79	6.20
1.52	1.38	3.87	4.54	5.12	5.15	5.50	5.92	4.56	2.46
6.90	1.47	2.11	2.32	6.75	5.84	8.80	7.40	4.72	3.62
2.46	8.75	2.65	7.86	4.71	6.25	9.45	12.80	1.42	1.92
7.60	8.79	5.92	9.65	5.09	4.11	6.37	5.40	11.25	3.90
5.33	8.64	7.41	7.95	10.60	3.81	3.78	3.75	3.10	6.43
1.70	6.40	3.24	1.79	4.90	3.49	6.77	5.62	9.70	5.11
4.50	2.50	5.21	1.76	9.20	1.20	6.85	2.80	7.35	11.75

Here is a histogram of the data and a quantile-quantile plot under a Weibull assumption:





(a) Do you think the Weibull model is a reasonable choice for the population-level distribution of ignition time? Explain.

(b) When I fit a Weibull(β, η) distribution to these data, R reported the maximum likelihood estimates

$$\widehat{\beta} = 2.05$$
$$\widehat{\eta} = 5.91.$$

Use these values to estimate the proportion of upholstery pieces in the population that will ignite before 5 seconds. *Hint:* I'm asking you to find $P(T < 5)$ using $\widehat{\beta}$ and $\widehat{\eta}$ as estimates of β and η, respectively.

(c) What does the survivor function $S_T(t)$ represent in this example? Explain what $S_T(t)$ is in words.

The next page is blank if you would like to use it for your answers to Problem 7.

This is a blank page for your answers to Problem 7. Use it if you wish.

Binomial:

$$p_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, 2, ..., n \\ 0, & \text{otherwise.} \end{cases}$$

Geometric:

$$p_X(x) = \begin{cases} (1-p)^{x-1} p, & x = 1, 2, 3, ... \\ 0, & \text{otherwise.} \end{cases}$$

Negative binomial:

$$p_X(x) = \begin{cases} \binom{x-1}{r-1} (1-p)^{x-r} p^r, & x = r, r+1, r+2, ..., \\ 0, & \text{otherwise.} \end{cases}$$

Hypergeometric:

$$p_X(x) = \begin{cases} \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}, & x \le K \text{ and } n - x \le N - K \\ 0, & \text{otherwise.} \end{cases}$$

Poisson:

$$p_X(x) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!}, & x = 0, 1, 2, ... \\ 0, & \text{otherwise.} \end{cases}$$

Exponential:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases} \qquad F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-\lambda x}, & x > 0. \end{cases}$$

Gamma:

$$f_X(x) = \begin{cases} \dfrac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Normal (Gaussian):

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \, e^{-(x-\mu)^2/2\sigma^2}, \quad \text{for } -\infty < x < \infty.$$

Weibull:

$$f_T(t) = \begin{cases} \dfrac{\beta}{\eta} \left(\dfrac{t}{\eta}\right)^{\beta-1} \exp\left[-\left(\dfrac{t}{\eta}\right)^{\beta}\right], & t > 0 \\ 0, & \text{otherwise.} \end{cases} \qquad F_T(t) = \begin{cases} 0, & t \le 0 \\ 1 - \exp\left[-\left(\dfrac{t}{\eta}\right)^{\beta}\right], & t > 0. \end{cases}$$