1. (a) This is the same as asking how many ways are there to permute 6 distinct objects (the cities). To "permute" means to arrange in a particular order. There are

$$6! = 720 \text{ possible itineraries.}$$

(b) This means each itinerary has the same probability (chance) of being selected. Therefore, each itinerary has probability 1/720 of being selected.
(c) In part (a), we calculated $n_S = 720$, the number of possible itineraries. Define the event

$$A = \{\text{West coast and East coast cities grouped together}\}.$$

How many itineraries satisfy this event? That is, what is n_A? Think of one itinerary satisfying A as follows:

$$(\_\_\ \_\_\ \_\_\ \ /\ \ \_\_\ \_\_\ \_\_).$$

We can determine n_A using the multiplication rule of counting:

- there are $n_1 = 2$ ways to order the coasts (visit West coast cities first or visit East coast cities first)
- there are $n_2 = 3!$ ways to permute the West coast cities
- there are $n_3 = 3!$ ways to permute the East coast cities.

There are

$$n_A = 2 \times 3! \times 3! = 72$$

itineraries satisfying the event A. Assuming each itinerary is equally likely,

$$P(A) = \frac{n_A}{n_S} = \frac{72}{720} = \frac{1}{10}.$$

2. (a) Define the events

$$\begin{aligned} A &= \{\text{bid prepared by Engineer 1}\} \\ B &= \{\text{bid contains an error}\}. \end{aligned}$$

We are given $P(A) = 0.70$, $P(B|A) = 0.02$, and $P(B|A') = 0.04$.
(b) Use the Law of Total Probability. The proportion of all bids that will contain an error is

$$\begin{aligned} P(B) &= P(B|A)P(A) + P(B|A')P(A') \\ &= 0.02(0.70) + 0.04(0.30) \\ &= 0.026. \end{aligned}$$

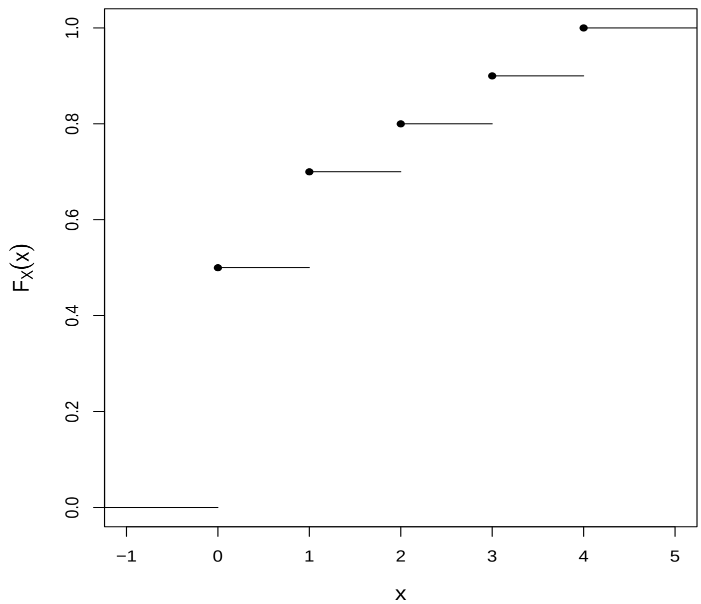
(c) We are told that the bid contains an error so B has occurred. We want $P(A|B)$. We can use Bayes' Rule to get this:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')} = \frac{0.02(0.70)}{0.026} = \frac{7}{13} \approx 0.538.$$

3. (a) The random variable X is discrete because it has a finite number of possible values. We can list them out: 0, 1, 2, 3, and 4.
(b) The mean number of serious accidents per year is

$$\begin{aligned} E(X) &= \sum_{\text{all } x} x p_X(x) \\ &= 0(0.5) + 1(0.2) + 2(0.1) + 3(0.1) + 4(0.1) = 1.1. \end{aligned}$$

(c) The cdf of X is easy to prepare by hand, but I used R:



4. (a) Let X denote the number of defective devices out of the 10 tested. Under the Bernoulli trial assumptions (next part), we have $X \sim b(10, 0.03)$. We want

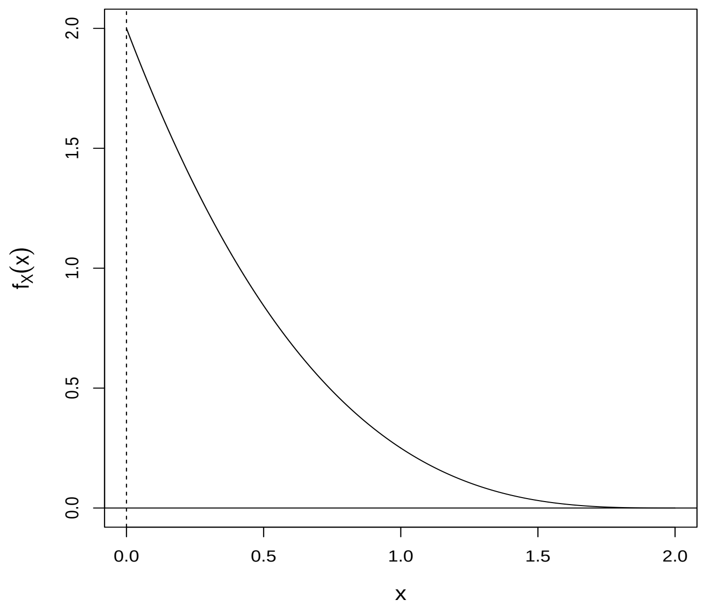
$$\begin{aligned} P(X = 1 \text{ or } 2) &= P(X = 1) + P(X = 2) \\ &= \binom{10}{1}(0.03)^1(0.97)^9 + \binom{10}{2}(0.03)^2(0.97)^8 \\ &= 10(0.03)(0.97)^9 + 45(0.0009)(0.97)^8 \approx 0.260. \end{aligned}$$

(b) The binomial distribution arises for X only when the Bernoulli trial assumptions hold. For this problem, they are

1. Each device is either defective or it is not.
2. The devices are independent. That is, the defective status of one device is not affected by the other devices.
3. The probability of a defective device, $p = 0.03$, is the same for all devices made by the manufacturer.

5. A graph of the pdf $f_X(x)$ is on the next page. In part (a), we want

$$P(X < 0.4) = \int_0^{0.4} f_X(x)dx = \int_0^{0.4} 0.25(2 - x)^3 dx.$$



To do the last integral, let

$$u = 2 - x \implies du = -dx.$$

With this u-substitution (noting the change in limits), the last integral equals

$$\int_{2}^{1.6} 0.25u^3(-du) = 0.25\int_{1.6}^{2} u^3 du = 0.25\left(\frac{u^4}{4}\right)\Big|_{1.6}^{2} = \frac{1}{16}\left[2^4 - (1.6)^4\right] = 1 - \frac{(1.6)^4}{16} \approx 0.590.$$

Therefore,

$$P(X < 0.4) \approx 0.590.$$

(b) We want to find the supply c so that

$$P(X > c) = 0.01 \iff P(X < c) = 0.99.$$

That is, c is the 99th percentile of the distribution of X. We have

$$P(X < c) = \int_0^c f_X(x)dx = \int_0^c 0.25(2-x)^3 dx.$$

Use the same u-substitution as in part (a). The last integral equals

$$\int_{2}^{2-c} 0.25u^3(-du) = 0.25\int_{2-c}^{2} u^3 du = 0.25\left(\frac{u^4}{4}\right)\Big|_{2-c}^{2} = \frac{1}{16}\left[2^4 - (2-c)^4\right] = 1 - \frac{(2-c)^4}{16}.$$

Now, set

$$1 - \frac{(2-c)^4}{16} \overset{\text{set}}{=} 0.99$$

and solve for c. We have

$$\frac{(2-c)^4}{16} = 0.01 \implies (2-c)^4 = 0.16 \implies 2 - c = (0.16)^{1/4} \implies c = 2 - (0.16)^{1/4} \approx 1.3675.$$

The capacity should be set at approximately 13,675 gallons.

6. (a) Let X denote the number of cars entering the tunnel per hour. We assume $X \sim$ Poisson($\lambda = 10$). We want

$$\begin{aligned} P(X \leq 2) &= P(X=0) + P(X=1) + P(X=2) \\ &= \frac{10^0 e^{-10}}{0!} + \frac{10^1 e^{-10}}{1!} + \frac{10^2 e^{-10}}{2!} \\ &= e^{-10}\left(1 + 10 + \frac{100}{2}\right) \\ &\approx 0.0028. \end{aligned}$$

(b) Let T denote the time until the first car enters the tunnel so that $T \sim$ exponential($\lambda = 10$). We want

$$P(T > 0.5) = 1 - P(T \leq 0.5) = 1 - F_T(0.5),$$

where $F_T(0.5)$ is the exponential($\lambda = 10$) cdf evaluated at $t = 0.5$. We have

$$1 - F_T(0.5) = 1 - [1 - e^{-10(0.5)}] = e^{-5} \approx 0.0067.$$

7. (a) The Weibull distribution looks like a good model for the ignition time. The histogram of the data has the typical "skewed-right" shape we would expect from the Weibull pdf, and the qq plot shows very good agreement between the observed ignition times and the Weibull quantiles.
(b) We want to estimate

$$P(T < 5) = F_T(5) = 1 - \exp\left[-\left(\frac{5}{\eta}\right)^{\beta}\right]$$

using $\widehat{\beta}$ and $\widehat{\eta}$ as estimates of β and η. We have

$$1 - \exp\left[-\left(\frac{5}{5.91}\right)^{2.05}\right] \approx 0.508.$$

Therefore, we would estimate about 50.8% of the upholstery pieces would ignite before 5 seconds.
(c) The survivor function

$$S_T(t) = P(T > t)$$

would represent the proportion of upholstery pieces in the population whose ignition time is larger than t.