1. (a) The population is all sprinkler systems (of this new type) manufactured by Johnson Controls.

(b) From Chapter 6, we know when the population distribution is normal, the sample mean $\overline{X}$ is also normally distributed. Specifically,

$$\overline{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right) \longrightarrow \mathcal{N}\left(130, \frac{2.25}{20}\right) \longrightarrow \mathcal{N}\left(130, 0.1125\right).$$

(c) The standard error of $\overline{X}$ is the square root of its variance, that is,

$$\text{se}(\overline{X}) = \sqrt{\frac{2.25}{20}} = \sqrt{0.1125} \approx 0.335.$$

The standard error of $\overline{X}$ measures how variable its value will be from sample to sample.

(d) The confidence interval gives values of the population mean μ that are consistent with the sample. If the 99% confidence interval is $(134.5, 136.0)$, this means we are 99% confident the population mean μ is between 134.5 and 136.0 deg F. This interval of values is not consistent with Johnson Controls' claim that the population mean is 130 deg F.

2. (a) The boxplots do not show what I would expect to see if the population variances were really equal (i.e., $\sigma_1^2 = \sigma_2^2$). The variation in the female sample is noticeably less than that of the male sample. I will use the confidence interval for $\Delta = \mu_1 - \mu_2$ that does not assume equal population variances.

We could write a confidence interval for the population variance ratio

$$\Lambda = \frac{\sigma_2^2}{\sigma_1^2}$$

if we wanted to perform statistical inference for the population variances. This interval uses the F distribution.

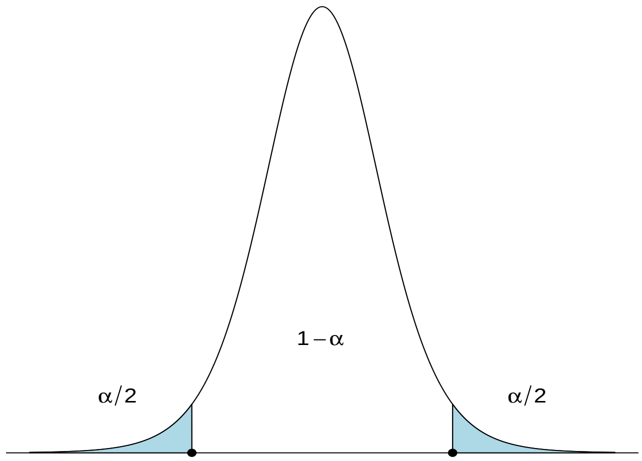
(b) We are 95% confident the population mean difference $\Delta = \mu_1 - \mu_2$ is between 0.88 and 2.28 micrograms per dL. Because this interval contains only positive values, this suggests (at the 95% confidence level) the population mean lead level for male workers (μ_1) is larger than the population mean lead level for female workers (μ_2).

(c) There looks to be some disagreement in the qq plot for the males. The larger lead levels are consistently larger than what they should be under a normal distribution. We see good agreement with the normal quantiles for the female sample.

Even though there is disagreement for the male sample, we know that statistical inference procedures for means are generally robust to normality violations. This means the intervals we calculate, like in part (b), are still meaningful. The large sample sizes help. Robustness for means inference is a consequence of the CLT, a result that applies for large samples.

3. (a) The population is all legally registered automobiles in Connecticut. The sample is the 200 automobiles selected.

(b) The value $z_{\alpha/2}$ is the upper $\alpha/2$ quantile of the standard normal distribution. This is shown as a dark circle in the picture below—the larger one on the right.



(c) There is a formula for the sample size n to estimate a population proportion. If you forgot it, you can derive it from the information in the question. The engineer wants the margin of error

$$z_{\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} = 0.01.$$

We have $z_{\alpha/2} = z_{0.01/2} \approx 2.58$ for a 99% confidence level. Also, the sample proportion of cars that passed the state's emission test is

$$\widehat{p} = \frac{124}{200} = 0.62.$$

Therefore,

$$\begin{aligned}
2.58\sqrt{\frac{0.62(1-0.62)}{n}} = 0.01 \quad &\Longrightarrow \quad \sqrt{\frac{0.2356}{n}} = \frac{0.01}{2.58} \\
&\Longrightarrow \quad \frac{0.2356}{n} = \left(\frac{0.01}{2.58}\right)^2 \\
&\Longrightarrow \quad \frac{n}{0.2356} = \left(\frac{2.58}{0.01}\right)^2 \\
&\Longrightarrow \quad n = 0.2356\left(\frac{2.58}{0.01}\right)^2 \approx 15682.48.
\end{aligned}$$

She would have to sample 15,683 cars to achieve 99% confidence and a margin of error equal to 0.01. This may not be feasible. It would depend on what resources she has and how easy it is to observe cars' emissions test status. To reduce the number of cars she needs to sample, she could

1. reduce the confidence level, for example, to 95%.
2. increase the margin of error.

4. (a) This is a matched pairs study because we get to see two gasoline consumption observations on each individual (car). This makes the two samples of gasoline consumption (Radial and Belted) dependent.

(b) (i) Recall we analyze data from matched pairs studies using the data differences. This means we calculate

$$\begin{aligned} \text{Car 1:} &\quad 4.2 - 4.1 = 0.1 \\ \text{Car 2:} &\quad 4.7 - 4.9 = -0.2 \\ \text{Car 3:} &\quad 6.6 - 6.2 = 0.4, \end{aligned}$$

and so on. Here, `diff` refers to the 12 data differences.

(ii) I will interpret both intervals (you only had to do one).

First interval: We are 95% confident the population mean difference $\Delta = \mu_1 - \mu_2$ is between 0.02 and 0.27 km/L. Because this interval contains only positive values, this suggests (at the 95% confidence level) the population mean gasoline consumption for radial tires (μ_1) is larger than the population mean gasoline consumption for belted tires (μ_2).

Second interval: We are 99% confident the population mean difference $\Delta = \mu_1 - \mu_2$ is between -0.04 and 0.32 km/L. Because this interval contains negative values and positive values, we cannot conclude (at the 99% confidence level) that one population mean gasoline consumption is larger or smaller than the other.

(c) Larger confidence levels will produce wider (longer) intervals. For these data, the 99% confidence interval is wide enough to include "0," which corresponds to the population means being equal. However, the 95% confidence interval, which will not be as wide, misses "0" on the high side. Therefore, at the 95% confidence level, we have evidence the population means are different. At the 99% confidence level, we don't.

5. (a) Let μ_1, μ_2, and μ_3 denote the population mean strengths for Brand 1, Brand 2, and Brand 3 boxes, respectively. The F statistic tests

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

versus

$$H_1 : \text{the population means are not all equal.}$$

The null hypothesis says the population mean strengths are equal for the three brands. The alternative hypothesis says that at least one population mean is different than the others.

The F statistic for these data $F = 16.29$ is very large. We know that if H_0 is true, then F should be around 1. This large value of F is more consistent with H_1 being true.

This can also be seen by the very small p-value $= 0.000012$. H_0 would be rejected at commonly used significance levels like $\alpha = 0.05$ or even $\alpha = 0.01$.

We conclude that at least one population mean strength is different than the other population means.

(b) If H_0 was true, then MS_T and MS_E would both estimate σ^2 unbiasedly. However, our analysis in part (a) suggests H_1 is more believable. Therefore, only MS_E is an unbiased estimator of σ^2. We would report

$$\text{MS}_\text{E} = 165.4$$

as an estimate of the common population variance σ^2. The data are measured in psi, so the variance estimate is measured in $(\text{psi})^2$.

(c) (i) This means we are 95% confident that all three of the intervals will contain their pairwise population mean difference. The "95% confidence" part refers to the group of intervals−not the individual intervals.

(ii) We see that both confidence intervals involving Brand 1, that is, for the pairwise differences

$$\mu_2 - \mu_1 \quad \text{and} \quad \mu_3 - \mu_1,$$

contain only positive values: $(12.37, 38.13)$ and $(13.72, 39.48)$. This suggests the population mean strengths μ_2 and μ_3 are both larger than μ_1. However, the confidence interval for

$$\mu_3 - \mu_2,$$

which compares Brand 2 and Brand 3, contains both negative values and positive values: $(-11.53, 14.23)$. From this interval, we cannot say that one population mean strength is larger or smaller than the other.

Therefore, we would conclude the population mean strengths of Brand 2 and Brand 3 are both larger than the population mean strength of Brand 1. However, we cannot say that Brand 2 and Brand 3 population means are different. To maximize population mean strength, our recommendation would be to use either Brand 2 or Brand 3 boxes.