GROUND RULES:

- Print your name at the top of this page. Do not put your name on any other page.
- This is a closed-book and closed-notes exam. A list of discrete and continuous distributions appears at the end.
- This exam contains 6 problems, each worth 12 points. This exam is worth **72 points** total.
- You may use a calculator, but this calculator cannot have internet access. You cannot use your phone as a calculator. You cannot share calculators with another student. Show all of your work; use a calculator only to do final calculations or to check your work.
- Each problem contains parts. On each part, there is opportunity for partial credit, so show all of your work and explain all of your reasoning. **Translation:** No work/no explanation means no credit.
- On any problem, you may use the back of the page if you need more space. I also have extra paper if you need it.
- Any discussion or inappropriate communication between you and another examinee, as well as the appearance of any unnecessary material, will result in a declaration of academic dishonesty. Don't risk it!
- You have 75 minutes to complete this exam.

HONOR PLEDGE FOR THIS EXAM:

After you have finished the exam, please read the following statement and sign your name below it.

> *I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own.*

1. A computer system uses passwords that are exactly **six** characters. Each character is one of the 26 letters (a–z) or 10 integers (0–9). Uppercase letters are not used.

(a) How many passwords are possible?

(b) What does it mean when we say "each possible password is equally likely?"

(c) Suppose you pick one password at random from all possible passwords. Calculate the probability of

$$A = \{\text{password contains 6 distinct letters}\}.$$

"Distinct" means that all 6 letters must be different.

Part (d) is on the next page.

(d) Suppose you pick one password at random from all possible passwords. Calculate the probability of

$$B = \{\text{password contains exactly 3 letters and exactly 3 numbers}\}.$$

The letters and numbers need not be distinct.

2. Laboratory glass is shipped in small packages or large packages.

- Among all shipments in small packages, 2% will be damaged.
- Among all shipments in large packages, 1% will be damaged.
- Forty percent (40%) of all shipments use small packages. The remaining shipments use large packages.

(a) Define two events A and B and write each number above using our notation for probability and conditional probability. Do **not** define more than two events (aside from complements) or you will make the problem too hard.

(b) Find the percentage of all shipments which are not damaged.

(c) Among all damaged shipments, what percentage use small packages?

3. An engineer selects 5 water specimens from lakes in the Atlanta area and records

$$X = \text{the number of specimens containing an industrial pollutant.}$$

The probability mass function (pmf) for X is shown below:

x	0	1	2	3	4	5
$p_X(x)$	0.10	0.25	0.30	0.20	0.10	0.05

(a) Is X a discrete or continuous random variable? Explain.

(b) Find the mean and variance of X.

Part (c) is on the next page.

(c) Prepare a graph of $F_X(x)$, the cumulative distribution function of X. Label both axes and use appropriate tick marks on each axis. Neatness counts.

4. In a recent study, epidemiologists used the Poisson distribution to model X, the number of asthma emergency department (ED) visits per day. For the zip codes studied (all in Seattle, WA), the mean number of visits was $\lambda = 1.8$ per day.

(a) Calculate the probability there are at least 2 asthma ED visits on a given day.

(b) What is the distribution of the **time** until the first asthma ED visit? the second? the tenth?

Part (c) is on the next page.

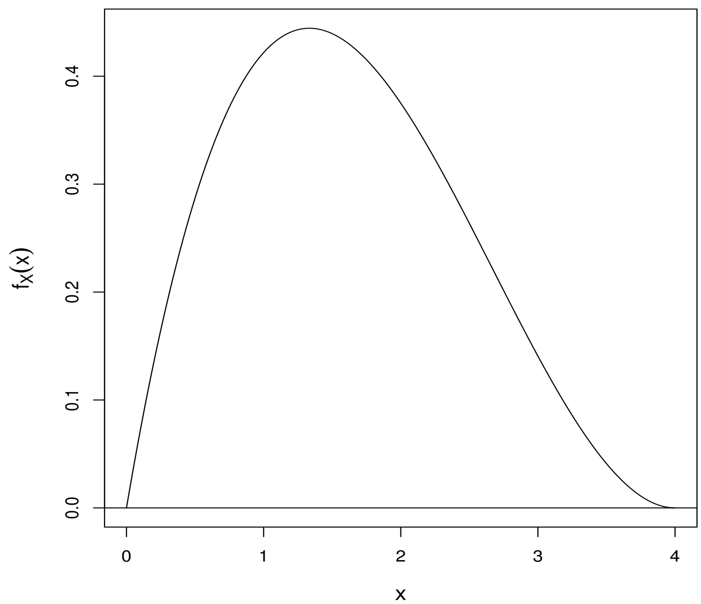
(c) In a one-month period (31 days), let Y denote the number of days where there are at least 2 asthma ED visits. What assumptions would be needed for Y to have a binomial distribution? There are three of them.

Hint: For part (c) only, think of each day as a "trial" and having at least 2 asthma ED visits in a day as a "success." You calculated the "success probability" in part (a).

5. A seismologist models the magnitude of "minor" earthquakes in South Carolina as a continuous random variable X with the following probability density function (pdf):

$$f_X(x) = \begin{cases} \frac{3x}{64}(4-x)^2, & 0 < x < 4 \\ 0, & \text{otherwise.} \end{cases}$$

The graph of $f_X(x)$ is shown below:



(a) What percentage of "minor" earthquakes will have magnitude less than 2?

Parts (b) and (c) are on the next page.

(b) Find the mean magnitude $E(X)$.

(c) Write an equation (involving an integral) that, when solved, will give the 90th percentile $\phi_{0.9}$ of the distribution. Your equation should have $\phi_{0.9}$ somewhere in it.

Note: You **don't** have to solve the equation to get a numerical answer. This would involve finding the roots of a quartic (degree 4) polynomial. Your calculators can probably do this, but you don't have to.

6. Nuclear power plants use heat exchangers to transfer energy from the reactor to steam turbines. A typical heat exchanger contains tubes through which steam flows continuously while the heat exchanger is in service. Engineers use the Weibull distribution to model

$$T = \text{time (in years) until a tube fails.}$$

Specifically, they assume the population of all tubes has a lifetime distribution which is Weibull with $\beta = 2.5$ and $\eta = 5.5$.

(a) What proportion of tubes in the population will fail before 3 years?

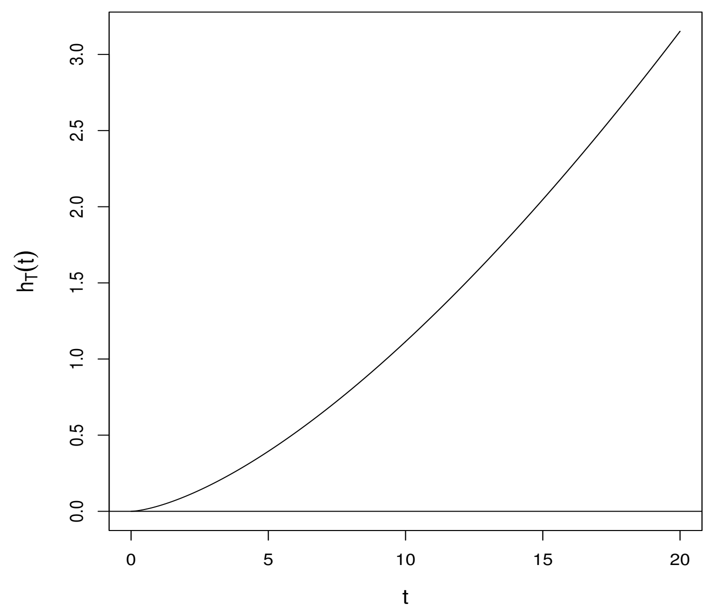
(b) Find the median time to failure. Another name for the median is the 50th percentile (or 0.5 quantile).

Parts (c) and (d) are on the next page.

(c) The hazard function of T, which is

$$h_T(t) = \frac{2.5}{5.5}\left(\frac{t}{5.5}\right)^{1.5},$$

is shown below. What does this graph tell us?



(d) One of the engineers is concerned the Weibull distribution does not do an adequate job of describing the failure times for this population of tubes. What is another lifetime distribution she could use?

Binomial:

$$p_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, 2, ..., n \\ 0, & \text{otherwise.} \end{cases}$$

Geometric:

$$p_X(x) = \begin{cases} (1-p)^{x-1} p, & x = 1, 2, 3, ... \\ 0, & \text{otherwise.} \end{cases}$$

Negative binomial:

$$p_X(x) = \begin{cases} \binom{x-1}{r-1} (1-p)^{x-r} p^r, & x = r, r+1, r+2, ..., \\ 0, & \text{otherwise.} \end{cases}$$

Hypergeometric:

$$p_X(x) = \begin{cases} \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}, & x \le K \text{ and } n - x \le N - K \\ 0, & \text{otherwise.} \end{cases}$$

Poisson:

$$p_X(x) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!}, & x = 0, 1, 2, ... \\ 0, & \text{otherwise.} \end{cases}$$

Exponential:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases} \qquad F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-\lambda x}, & x > 0. \end{cases}$$

Gamma:

$$f_X(x) = \begin{cases} \dfrac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Normal (Gaussian):

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, \quad \text{for } -\infty < x < \infty.$$

Weibull:

$$f_T(t) = \begin{cases} \dfrac{\beta}{\eta}\left(\dfrac{t}{\eta}\right)^{\beta-1} \exp\left[-\left(\dfrac{t}{\eta}\right)^{\beta}\right], & t > 0 \\ 0, & \text{otherwise.} \end{cases} \qquad F_T(t) = \begin{cases} 0, & t \le 0 \\ 1 - \exp\left[-\left(\dfrac{t}{\eta}\right)^{\beta}\right], & t > 0. \end{cases}$$