2.71. (a)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.3} = \frac{1}{3}$$

(b)

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.5} = 0.2$$

(c) Note that $A \subset A \cup B$. Therefore, $A \cap (A \cup B) = A$. Therefore,

$$P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B) - P(A \cap B)} = \frac{0.5}{0.5 + 0.3 - 0.1} = \frac{5}{7}$$

(d) Note that $A \cap B \subset A$. Therefore, $(A \cap B) \cap A = A \cap B$. Therefore,

$$P(A|A \cap B) = \frac{P(A \cap (A \cap B))}{P(A \cap B)} = \frac{P(A \cap B)}{P(A \cap B)} = 1.$$

This makes sense intuitively; if the outcome is in $A \cap B$ (which is given), then it is in A.
(e) Note that $A \cap B \subset A \cup B$. Therefore, $(A \cap B) \cap (A \cup B) = A \cap B$. Therefore,

$$P(A \cap B|A \cup B) = \frac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} = \frac{P(A \cap B)}{P(A \cup B)} = \frac{0.1}{0.5 + 0.3 - 0.1} = \frac{1}{7}$$

2.76. Define the following events:

$$\begin{aligned} D &= \{\text{consumer dissatisfied}\} \\ A &= \{\text{plumber A does the job}\}. \end{aligned}$$

We are given $P(D) = 0.1$, $P(A|D) = 0.5$, and $P(A) = 0.4$.
(a) We want $P(D|A)$. Note that

$$P(D|A) = \frac{P(A \cap D)}{P(A)} = \frac{P(A|D)P(D)}{P(A)} = \frac{0.5(0.1)}{0.4} = 0.125.$$

(b) We want $P(\overline{D}|A)$. Using the complement rule for conditional probabilities,

$$P(\overline{D}|A) = 1 - P(D|A) = 1 - 0.125 = 0.875.$$

2.81. Suppose $P(A) > 0$ and $P(B) > 0$. We are also given $P(A) < P(A|B)$. This means

$$P(A) < \frac{P(A \cap B)}{P(B)} \iff P(A)P(B) < P(A \cap B).$$

Therefore,

$$P(B|A) = \frac{P(A \cap B)}{P(A)} > \frac{P(A)P(B)}{P(A)} = P(B),$$

as claimed.

2.82. Suppose $P(A) > 0$ and $P(B) > 0$. We are given $A \subset B$. This means $A \cap B = A$. Now,

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1.$$

Also,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}.$$

2.114. Suppose suspect 1 is guilty and will lie on the lie detector test. Suppose suspect 2 is not guilty and will tell the truth on the lie detector test. Define the following events:

$$\begin{aligned} A_1 &= \{\text{lie detector shows positive reading (i.e., lying) for suspect 1}\} \\ A_2 &= \{\text{lie detector shows positive reading (i.e., lying) for suspect 2}\}. \end{aligned}$$

We are given $P(A_1) = 0.95$ because suspect 1 is lying and $P(A_2) = 0.10$ because suspect 2 tells the truth. Because the lie detector operates independently, we assume A_1 and A_2 are **independent** events.
(a) We want $P(A_1 \cap A_2)$. Because A_1 and A_2 are independent,

$$P(A_1 \cap A_2) = P(A_1)P(A_2) = 0.95(0.10) = 0.095.$$

(b) We want $P(A_1 \cap \overline{A}_2)$. Because complements of independent events are also independent,

$$P(A_1 \cap \overline{A}_2) = P(A_1)P(\overline{A}_2) = 0.95(1 - 0.10) = 0.855.$$

(c) We want $P(\overline{A}_1 \cap A_2)$. Because complements of independent events are also independent,

$$P(\overline{A}_1 \cap A_2) = P(\overline{A}_1)P(A_2) = (1 - 0.95)(0.10) = 0.005.$$

(d) We want $P(A_1 \cup A_2)$. By the additive rule,

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.95 + 0.10 - (0.95)(0.10) = 0.955.$$

2.117. Team 1 always passes cars; Team 2 always rejects cars. Define the following events:

$$\begin{aligned} A_1 &= \{\text{auto 1 is rejected}\} \\ A_2 &= \{\text{auto 2 is rejected}\} \\ A_3 &= \{\text{auto 3 is rejected}\} \\ A_4 &= \{\text{auto 4 is rejected}\}. \end{aligned}$$

We are given $P(A_1) = P(A_2) = P(A_3) = P(A_4) = 0.5$ as the two inspection teams are randomly assigned (i.e., Team 1 and Team 2 are assigned with the same probability). Although it does not say specifically in the question, we are supposed to assume the status of the cars' inspections are mutually independent.
(a) Define the event

$$A = \{\text{three of 4 cars are rejected}\}.$$

Note that we can write A as

$$A = (A_1 \cap A_2 \cap A_3 \cap \overline{A}_4) \cup (A_1 \cap A_2 \cap \overline{A}_3 \cap A_4) \cup (A_1 \cap \overline{A}_2 \cap A_3 \cap A_4) \cup (\overline{A}_1 \cap A_2 \cap A_3 \cap A_4)$$

These four events are mutually exclusive; therefore,

$$P(A) = P(A_1 \cap A_2 \cap A_3 \cap \overline{A}_4) + P(A_1 \cap A_2 \cap \overline{A}_3 \cap A_4) + P(A_1 \cap \overline{A}_2 \cap A_3 \cap A_4) + P(\overline{A}_1 \cap A_2 \cap A_3 \cap A_4).$$

Because A_1, A_2, A_3, and A_4 are mutually independent, the first probability

$$\begin{aligned} P(A_1 \cap A_2 \cap A_3 \cap \overline{A}_4) = P(A_1)P(A_2)P(A_3)P(\overline{A}_4) &= P(A_1)P(A_2)P(A_3)[1 - P(A_4)] \\ &= 0.5(0.5)(0.5)(1 - 0.5) = (0.5)^4. \end{aligned}$$

The three other probabilities are the same. Therefore,

$$P(A) = (0.5)^4 + (0.5)^4 + (0.5)^4 + (0.5)^4 = 4(0.5)^4 = 0.25.$$

(b) We have

$$P(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 \cap \overline{A}_4) = P(\overline{A}_1)P(\overline{A}_2)P(\overline{A}_3)P(\overline{A}_4) = (0.5)^4 = 0.0625.$$

2.128. (a) Suppose $P(A|B) = P(A|\overline{B})$. The LOTP says

$$P(A) = P(A|B)P(B) + P(A|\overline{B})P(\overline{B}).$$

Because $P(A|B) = P(A|\overline{B})$, we have

$$P(A) = P(A|B)P(B) + P(A|B)P(\overline{B}) = P(A|B)[P(B) + P(\overline{B})] = P(A|B),$$

as $P(B) + P(\overline{B}) = 1$. This means A and B are independent.
(b) Write $P(A)$ in its LOTP expansion; i.e.,

$$P(A) = P(A|C)P(C) + P(A|\overline{C})P(\overline{C}).$$

Similarly, write $P(B)$ in its LOTP expansion:

$$P(B) = P(B|C)P(C) + P(B|\overline{C})P(\overline{C}).$$

Suppose $P(A|C) > P(B|C)$ and $P(A|\overline{C}) > P(B|\overline{C})$ as stated in the problem. Then,

$$\begin{aligned} P(A) &= P(A|C)P(C) + P(A|\overline{C})P(\overline{C}) \\ &> P(B|C)P(C) + P(B|\overline{C})P(\overline{C}) = P(B). \end{aligned}$$

2.135. Define the following events:

$$\begin{aligned} A &= \{\text{person traveling for business reasons}\} \\ B_1 &= \{\text{traveling on major airline}\} \\ B_2 &= \{\text{traveling on private plane}\} \\ B_3 &= \{\text{traveling commercial on a non-major airline}\}. \end{aligned}$$

We are given $P(B_1) = 0.6$, $P(B_2) = 0.3$, and $P(B_3) = 0.1$. Note that $\{B_1, B_2, B_3\}$ partition the sample space of possible flight offerings at this small airport. We are also given $P(A|B_1) = 0.5$, $P(A|B_2) = 0.6$, and $P(A|B_3) = 0.9$.
(a) We want $P(A)$. Use LOTP:

$$\begin{aligned} P(A) &= P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3) \\ &= 0.5(0.6) + 0.6(0.3) + 0.9(0.1) = 0.57. \end{aligned}$$

(b) We want $P(A \cap B_2)$. Use the multiplication rule:

$$P(A \cap B_2) = P(A|B_2)P(B_2) = 0.6(0.3) = 0.18.$$

(c) We want $P(B_2|A)$. By Bayes' rule,

$$P(B_2|A) = \frac{P(A|B_2)P(B_2)}{P(A)} = \frac{0.6(0.3)}{0.57} \approx 0.316.$$

(d) I interpret this as wanting $P(A|B_1 \cup B_3) = P(A|\overline{B}_2)$ because both major airlines and commercial non-major airlines are commercially owned. We have

$$\begin{aligned} P(A|\overline{B}_2) = \frac{P(A \cap \overline{B}_2)}{P(\overline{B}_2)} = \frac{P(\overline{B}_2|A)P(A)}{P(\overline{B}_2)} &= \frac{[1 - P(B_2|A)]P(A)}{1 - P(B_2)} \\ &\approx \frac{(1 - 0.316)(0.57)}{1 - 0.3} \approx 0.557. \end{aligned}$$

2.158. Define the events

$$\begin{aligned} A &= \{\text{first ball white}\} \\ B &= \{\text{second ball black}\}. \end{aligned}$$

We want to compute $P(A|B)$. Use Bayes' Rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\overline{A})P(\overline{A})}.$$

Finding $P(A)$ is easy. We choose the first ball from a bowl with w white and b black balls. Therefore, assuming each ball is equally likely,

$$P(A) = \frac{w}{w+b}.$$

Next we find $P(B|A)$. If A occurs, then we know that a first ball was white. Therefore, n white balls are going to be added to the bowl for the second selection; i.e.,

$$P(B|A) = \frac{b}{w+b+n}.$$

The last to find is $P(B|\overline{A})$. If A does not occur, then we know that a first ball was black. Therefore, n black balls are going to be added to the bowl for the second selection; i.e.,

$$P(B|\overline{A}) = \frac{b+n}{w+b+n}.$$

Therefore,

$$\begin{aligned} P(A|B) = \frac{\left(\frac{b}{w+b+n}\right)\left(\frac{w}{w+b}\right)}{\left(\frac{b}{w+b+n}\right)\left(\frac{w}{w+b}\right) + \left(\frac{b+n}{w+b+n}\right)\left(1 - \frac{w}{w+b}\right)} &= \frac{\left(\frac{b}{w+b+n}\right)\left(\frac{w}{w+b}\right)}{\left(\frac{b}{w+b+n}\right)\left(\frac{w}{w+b}\right) + \left(\frac{b+n}{w+b+n}\right)\left(\frac{b}{w+b}\right)} \\ &= \frac{bw}{bw + (b+n)b} \\ &= \frac{w}{w+b+n}, \end{aligned}$$

as claimed.

2.163. Define the events

$$\begin{aligned} A_1 &= \{\text{circuit 1 functions properly}\} \\ A_2 &= \{\text{circuit 2 functions properly}\} \\ A_3 &= \{\text{circuit 3 functions properly}\} \\ A_4 &= \{\text{circuit 4 functions properly}\}. \end{aligned}$$

For design A, define

$$\begin{aligned} C &= \{\text{current flows past circuits 1 and 2}\} \\ D &= \{\text{current flows past circuits 3 and 4}\}. \end{aligned}$$

Note that C and D are independent events (i.e., C depends only on the first two circuits; D depends only on the last two circuits−no overlapping circuits between C and D). The probability current will flow for design A is therefore

$$P(C \cap D) = P(C)P(D).$$

Now,

$$\begin{aligned} P(C) = P(A_1 \cup A_2) &= P(A_1) + P(A_2) - P(A_1 \cap A_2) \\ &= P(A_1) + P(A_2) - P(A_1)P(A_2) \\ &= 0.9 + 0.9 - (0.9)(0.9) = 0.99. \end{aligned}$$

$P(D) = 0.99$ by the same calculation (with circuits 3 and 4). Therefore, $P(C \cap D) = 0.99 \times 0.99 = 0.9801$.

For design B, here are all the instances where current will flow:

Circuits working	Probability
1,3	0.9(0.1)(0.9)(0.1)
2,4	0.1(0.9)(0.1)(0.9)
1,2,3	0.9(0.9)(0.9)(0.1)
1,2,4	0.9(0.9)(0.1)(0.9)
1,3,4	0.9(0.1)(0.9)(0.9)
2,3,4	0.1(0.9)(0.9)(0.9)
1,2,3,4	0.9(0.9)(0.9)(0.9)

All of these instances are mutually exclusive, so we add the corresponding probabilities; i.e.,

$$2(0.9)^2(0.1)^2 + 4(0.9)^3(0.1) + (0.9)^4 = 0.9639.$$

Therefore, design A has a higher probability; i.e., it is more reliable.