

1. Compute the following limits:

$$(a) \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} \quad (b) \lim_{x \rightarrow 0} \frac{x}{\sin x} \quad (c) \lim_{x \rightarrow \infty} x^2 e^{-x}$$

2. Find all values of  $x$  satisfying  $f(x) = 0$ :

$$(a) f(x) = x^2 + 2x + 2$$

$$(b) f(x) = 2^{x+2} - 3^{x-1}$$

$$(c) f(x) = \ln(4x - 7)$$

3. Compute the following integrals:

$$(a) \int_0^1 x^2(1-x) dx \quad (b) \int_0^{10} \frac{1}{2} e^{-x/2} dx \quad (c) \int_1^e \frac{1}{x} dx$$

4. Find the derivatives of the following functions:

$$(a) f(x) = e^{ax^2+bx+c}$$

$$(b) g(u) = u \ln u - u$$

$$(c) h(y) = \ln(y^2 - 4y)$$

5. Compute the following derivatives:

$$(a) \frac{d^3}{dx^3} x e^x \quad (b) \frac{d^2}{dt^2} e^{t+2t^2}$$

6. Find the following sums:

$$(a) \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \quad (b) \sum_{j=1}^{\infty} \left(\frac{1}{10}\right)^j \quad (b) \sum_{j=1}^{\infty} \left(\frac{1}{10}\right)^{j-1} \quad (d) \sum_{y=1}^9 \frac{2}{3} \left(\frac{1}{3}\right)^{y-1}$$

7. Evaluate the following double integrals:

$$(a) \int_0^1 \int_0^x 2xy^2 dy dx \quad (b) \int_u^1 \int_{u/y_2}^1 1 dy_1 dy_2, \quad 0 < u < 1$$

8. Compute the following integrals:

$$(a) \int_0^{\infty} x e^{-x} dx \quad (b) \int_0^{\infty} x^2 e^{-x} dx \quad (c) \int_0^5 x^2 e^{-x/2} dx$$

9. Evaluate the following:

$$(a) \lim_{x \rightarrow 0} xe^x \quad (b) \lim_{x \rightarrow \infty} xe^x \quad (c) \lim_{x \rightarrow 0} xe^{-x} \quad (d) \lim_{x \rightarrow \infty} xe^{-x}$$

10. Suppose that  $f(x) = 4x^2 - 1$ . Write a formula for  $f^{-1}$ , the inverse of  $f$ . Graph  $f$  and  $f^{-1}$  on the same set of axes. Do the same with  $g(x) = e^{-x}$  and  $h(x) = \ln(x - 1)$ .

11. Compute the following limits:

$$(a) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \quad (b) \lim_{n \rightarrow \infty} \left(1 - \frac{2}{n}\right)^n$$

12. Discuss the increasing/decreasing behavior of the functions (a)  $f(x) = 1 - e^{-x/2}$  and (b)  $f(x) = x^3e^{-x}$ . Restrict attention to  $x > 0$ . Also, discuss concavity.

13. Compute the following sums:

$$(a) \sum_{j=0}^{\infty} \frac{2^j}{j!} \quad (b) \sum_{j=1}^{\infty} \frac{1}{j} \quad (c) \sum_{j=0}^{\infty} \left(\frac{|x|}{1 + |x|}\right)^j$$

14. Compute the following integrals:

$$(a) \int_0^{\infty} \int_x^{x+1} e^{-y} dy dx \quad (b) \int_0^1 \int_0^1 y_1 e^{-(y_1+y_2)} dy_2 dy_1$$

15. Find the derivatives of the following functions and evaluate their derivatives at  $t = 0$ :

$$(a) f(t) = (0.3 + 0.7e^t)^{10} \quad (b) f(t) = e^{10t+t^2/2}$$

16. Compute the following integrals:

$$(a) \int_0^1 e^x \sqrt{1 - e^x} dx \quad (b) \int_0^{\infty} y^3 e^{-y^4/2} dy$$

17. Find the Taylor Series expansion of  $f(x) = \sin x$  about  $x = 0$ . Repeat for  $g(x) = e^x$ .

18. Write  $(x + y)^6$  in its binomial expansion.

19. Find both partial derivatives of  $f(x, y) = x \ln(xy)$ . Also, compute the Hessian of  $f(x, y)$ ; i.e., the matrix of second partial derivatives.

20. Show that

$$\sum_{x=0}^{\infty} \frac{1}{(x+1)(x+2)} = 1 \quad \text{and} \quad \sum_{x=0}^{\infty} \frac{x}{(x+1)(x+2)} = +\infty$$