1. Suppose $Y_1, Y_2, ..., Y_n$ is an iid sample from a Poisson(θ) distribution, where $\theta > 0$ is unknown.
(a) Argue $\overline{Y}$ is a consistent estimator for θ.
(b) Find a consistent estimator of $g(\theta) = \ln \theta$.
(c) The third sample moment $m_3' = n^{-1}\sum_{i=1}^{n} Y_i^3$ converges in probability to what constant?

2. Do Problem 9.24 in WMS, parts (a) and (b). Add the following part:
(c) Find a function of W_n that converges in distribution to a standard normal distribution as $n \to \infty$. *Hint:* Use the CLT.

3. An engineering component has a lifetime Y which follows a shifted exponential distribution, in particular, the probability density function (pdf) of Y is

$$f_Y(y|\theta) = \begin{cases} e^{-(y-\theta)}, & y > \theta \\ 0, & \text{otherwise.} \end{cases}$$

The unknown parameter $\theta > 0$ measures the magnitude of the shift. From an iid sample of component lifetimes $Y_1, Y_2, ..., Y_n$, we would like to estimate θ.
(a) Show the pdf of $Y_{(1)}$, the minimum order statistic, is

$$f_{Y_{(1)}}(y) = \begin{cases} ne^{-n(y-\theta)}, & y > \theta \\ 0, & \text{otherwise.} \end{cases}$$

(b) Show that $Y_{(1)}$ is a consistent estimator of θ.

4. Suppose $Y_1, Y_2, ..., Y_n$ is an iid sample from a beta(α, β) population distribution, where $\alpha = \beta = \theta$, and $\theta > 0$ is unknown. That is, the population probability density function (pdf) is

$$f_Y(y) = \begin{cases} \dfrac{\Gamma(2\theta)}{[\Gamma(\theta)]^2}\, y^{\theta-1}(1-y)^{\theta-1}, & 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the method of moments (MOM) estimator of θ. *Hint:* $E(Y) = 1/2$ is free of θ, so equate second moments.
(b) Is the MOM estimator a consistent estimator of θ? Prove or disprove.

5. Use Monte Carlo simulation to approximate the following integrals:

(a) $\displaystyle\int_0^\infty \ln(y+1)ye^{-y}dy$ (b) $\displaystyle\int_{-\infty}^{\infty} (z^3+1)e^{-z^2/2}dz$ (c) $\displaystyle\int_0^1 x^3(1-x)^2 \sin x \, dx$