

GROUND RULES:

- Print your name at the top of this page. Do not put your name on any other page.
- This is a closed-book and closed-notes exam.
- This exam contains 6 problems, each worth 10 points. This exam is worth **60 points** total.
- You may use a calculator, but this calculator cannot have internet access. You cannot use your phone as a calculator. You cannot share calculators with another student. Show all of your work; use a calculator only to do final calculations or to check your work.
- Each problem contains parts. On each part, there is opportunity for partial credit, so show all of your work and explain all of your reasoning. **Translation:** No work/no explanation means no credit.
- On any problem, you may use the back of the page if you need more space. I also have extra paper if you need it.
- Any discussion or inappropriate communication between you and another examinee, as well as the appearance of any unnecessary material, will result in a declaration of academic dishonesty. Don't risk it!
- Table 1 is attached at the end if you need to do any calculations for a normal distribution.
- You have 55 minutes to complete this exam.

HONOR PLEDGE FOR THIS EXAM:

After you have finished the exam, please read the following statement and sign your name below it.

I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own.

1. An experiment consists of flipping a fair coin (H, T) and rolling a fair die (1, 2, 3, 4, 5, and 6). The combined coin-die outcome will be observed. For example, the outcome “H-3,” means the coin flip was a “head” and the die roll was “3.”

(a) Write out a sample space S for this experiment. How many outcomes are in your sample space?

(b) Define the events

$$A = \{\text{coin flip is T}\}$$

$$B = \{\text{die roll is an odd number}\}.$$

Check to see if A and B are independent. State any assumptions you are making.

2. Laboratory glass is shipped in small packages or large packages.

- Among all shipments in small packages, 5% will be damaged.
- Among all shipments in large packages, 15% will be damaged.
- Forty percent (40%) of all shipments use small packages. The remaining shipments use large packages.

Use the Law of Total Probability

$$P(A) = P(A|B)P(B) + P(A|\overline{B})P(\overline{B})$$

to find the percentage of all shipments which are damaged. Make sure you clearly define what the events A and B are in your solution.

3. The percentage of drivers in South Carolina who are currently driving with an expired license is 10%. Police have set up “check points” to find drivers who have an expired license. At one check point, a police officer observes 10 drivers. Define Y to be the number of drivers (out of 10) who have an expired drivers license.

(a) In this problem, what assumptions would be needed for Y to have a binomial distribution?

Hint: Recall the Bernoulli trial assumptions. What is a “trial” here? a “success?”

(b) If the assumptions in part (a) are satisfied, calculate the probability **at least two** drivers have an expired drivers license. Recall the $b(n, \pi)$ probability mass function is

$$p_Y(y) = \begin{cases} \binom{n}{y} \pi^y (1 - \pi)^{n-y}, & y = 0, 1, 2, \dots, n \\ 0, & \text{otherwise.} \end{cases}$$

Show your work here. Don't just write the answer (if your calculator can do it).

4. The brain weights of a population of Swedish males follow a normal distribution with mean $\mu = 1400$ grams and standard deviation $\sigma = 100$ grams.

(a) Prepare a graph of this normal distribution. Identify where the mean is, and also identify where $\mu \pm \sigma$, $\mu \pm 2\sigma$, and $\mu \pm 3\sigma$ are located.

(b) Let Y denote the brain weight (in grams) for individuals in this population. Rewrite the event

$$\{Y > 1550\}$$

in terms of a standard normal random variable Z .

(c) Calculate $y_{0.10}$, the 10th percentile of this distribution. Interpret what this value means. *Hint:* The 10th percentile of a standard normal distribution is $z_{0.10} \approx -1.28$, from Table 1.

5. In class, we learned that if $Y_1, Y_2, \dots, Y_n$ is a random sample from a $\mathcal{N}(\mu, \sigma^2)$ population distribution, then the sample mean $\bar{Y}$ also follows a normal distribution, specifically,

$$\bar{Y} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right).$$

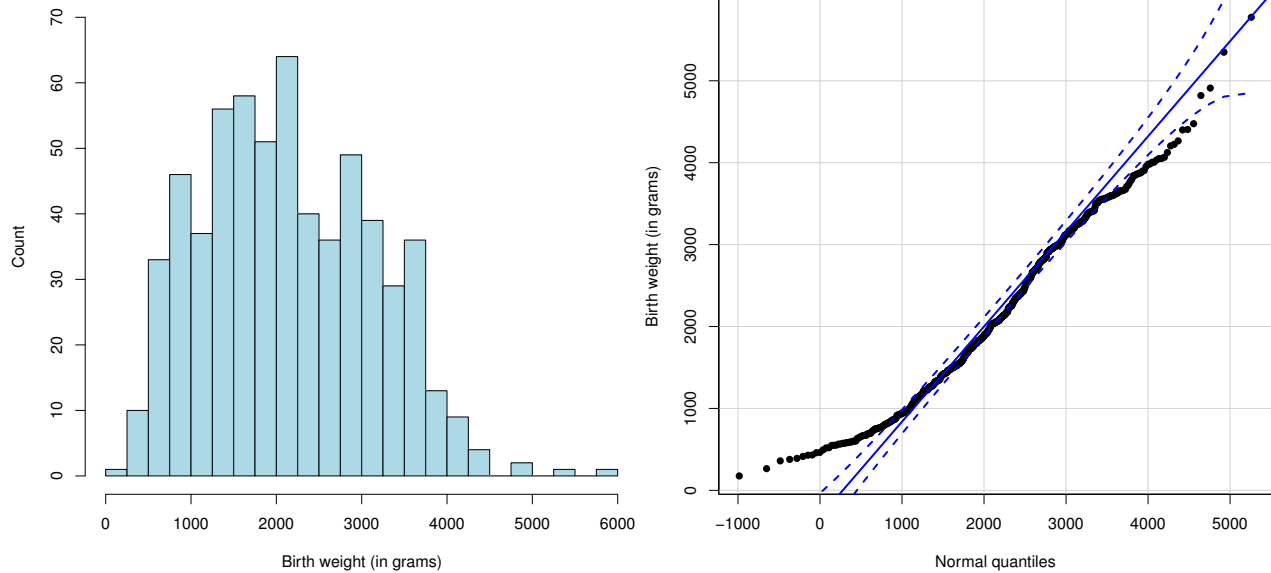
(a) Using the information from Problem 4, find the probability the sample mean of $n = 25$ brain weights will be less than 1350 grams.

(b) If the population distribution of brain weights was not normal (e.g., suppose it is skewed, etc.), what can we say about the sampling distribution of $\bar{Y}$?

6. In an 18-month period, there were 615 infants admitted to the neonatal intensive care unit at Palmetto Richland Hospital in Columbia, SC. The random variable

$$Y = \text{birth weight (in grams)}$$

was measured on each infant producing the large data set $y_1, y_2, \dots, y_{615}$. A histogram and normal quantile-quantile (qq) plot of the data are shown below.



(a) Do you think these data are well represented by a normal population distribution? Why or why not?

(b) In this problem, is Y best regarded as a discrete or continuous random variable? Explain.