STAT718A. HW2. SOL'NS

4. pre-shape $\underset{\sim}{z} = \underset{\sim}{x} + i\underset{\sim}{y}$, $k-1$ vector.

The pre-shapes which are furthest away in terms of ρ must be at a distance $\rho = \pi/2$, or equivalently $d_F = 1$ $(= \sin\rho)$.

From (3.8) in the notes/book we have

$$d_F(\underset{\sim}{\mu}, \underset{\sim}{z}) = \left\{ 1 - \frac{\underset{\sim}{z}^* \underset{\sim}{\mu} \underset{\sim}{\mu}^* \underset{\sim}{z}}{\underset{\sim}{z}^* \underset{\sim}{z} \underset{\sim}{\mu}^* \underset{\sim}{\mu}} \right\}^{1/2}$$

and so we require $\underset{\sim}{z}^* \underset{\sim}{\mu} = 0$ to give $d_F(\underset{\sim}{\mu}, \underset{\sim}{z}) = 1$.

Hence, the set of pre-shapes which are furthest away are

$$\{ \underset{\sim}{\mu} : \underset{\sim}{z}^* \underset{\sim}{\mu} = 0 \}.$$

5. $d_F = \sin\rho \qquad 0 \le \rho \le \pi/2$

$$d_P = \sqrt{2}\,(1-\cos\rho)^{1/2} \quad \Rightarrow \quad \cos\rho = 1 - d_P^2/2 .$$

<u>So</u>

$$\begin{aligned} d_F &= \sqrt{1-\cos^2\rho} \\ &= \sqrt{1-(1-d_P^2/2)^2} \\ &= \sqrt{1-(1-d_P^2+d_P^4/4)} \\ &= (d_P^2 - d_P^4/4)^{1/2} \\ &= d_P\left(1 - d_P^2/4\right)^{1/2} \end{aligned}$$

Consider a Taylor series expansion for $f(d_P) = \sqrt{1 - d_P^2/4}$ about zero.

$$f(d_P) = f(0) + f'(0)d_P + \frac{f''(0)}{2!}d_P^2 + \frac{f'''(0)}{3!}d_P^3 + \cdots$$

<u>Now</u> $\quad f'(d_P) = \frac{1}{2}\left(1-\frac{d_P^2}{4}\right)^{-1/2}\cdot -\frac{d_P}{2}$

$$f''(d_P) = -\frac{1}{2}\cdot\frac{1}{2}\left(1-\frac{d_P^2}{4}\right)^{1/2}\cdot - \frac{1}{2}\cdot\frac{d_P}{2}\cdot\frac{1}{2}\left(-\frac{1}{2}\right)\left(1-\frac{d_P^2}{4}\right)^{-3/2}$$

<u>So</u> $\quad f(0) = 1, \quad f'(0) = 0, \quad f''(0) = -1/4$

Hence $\quad f(d_P) = 1 - \dfrac{d_P^2}{8} + O(d_P^3)$

<u>But</u> we require a Taylor series expansion for d_F which is therefore given by.

$$d_F = d_P\, f(d_P) = d_P - d_P^3/8 + O(d_P^4) .$$