STAT 511 fa 2019 Lec 02 slides

Counting rules

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These slides are an instructional aid; their sole purpose is to display, during the lecture, definitions, plots, results, etc. which take too much time to write by hand on the blackboard. They are not intended to explain or expound on any material.

Motivation to study counting rules

If all outcomes in $\mathcal{S}$ are equally likely, for any event $A \subset \mathcal{S}$, we have

$$P(A) = \frac{\#\{\text{sample points in } A\}}{\#\{\text{sample points in } \mathcal{S}\}}.$$

This leads to an interest in counting rules.

Fundamental theorem of counting

If a job consists of K tasks such that the tasks may be completed in $n_1, \dots, n_K$ ways, respectively, then there are

$$\prod_{k=1}^{K} n_k = n_1 \times n_2 \times \cdots \times n_K$$

ways to do the job.

Exercise: In how many ways can you complete the following three tasks?

(i) choose to wear shorts or long pants.

(ii) choose one of three pairs of shoes.

(iii) choose to take the red pill or the blue pill.

Many counting situations are like drawing r things from N things...

I. ordered, without replacement
II. unordered, without replacement
III. ordered, with replacement
IV. unordered, with replacement

Exercise: Suppose \$15 is to be disbursed among 60 students, such that

1. the top 5 of the 60 students receive \$5, \$4, \$3, \$2, and \$1, respectively.
2. the first 5 students to enter the classroom will each receive \$3.
3. during the next 5 class periods, respectively, \$6.82, \$4.36, \$2.45, \$1.09, and \$0.28 will be given to a randomly selected student.
4. fifteen \$1 bills will be distributed amongst the 60 students.

Find the number of ways in which each of the above can be done.

The numbers of ways to draw r things from N things are:

	Without replacement	With replacement
Orderered	$\dfrac{N!}{(N-r)!}$	N^r
Unordered	$\dbinom{N}{r}$	$\dbinom{N+r-1}{r}$

Exercise: Consider drawing 5 cards from a 52-card deck. Find the number of

1. hands with the ace of diamonds.
2. full house hands (three cards of one rank and two of another).
3. flush hands (5 cards of the same suit but not sequential).
4. straight hands (5 cards in sequence but not all of the same suit).

Then compute the corresponding probabilities.

Number of ways to partition

The number of ways to partition N things into K groups of sizes $n_1, \dots, n_K$, where $n_1 + \cdots + n_K = N$, is

$$\frac{N!}{n_1! \times \cdots \times n_K!}.$$

Exercise: Suppose 12 passengers will ride in 3 vehicles taking 4, 5, and 3 passengers, respectively.

1. In how many ways can the passengers be assigned to the different vehicles?
2. Suppose you and a friend are among the passengers. If seats are assigned at random such that every possible assignment to vehicles is equally likely, what is the probability that you will ride in a vehicle with your friend?