# STAT 511 fa 2019 Lec 07 slides

# Suite of ought-to-know probability distributions of discrete random variables

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These slides are an instructional aid; their sole purpose is to display, during the lecture, definitions, plots, results, etc. which take too much time to write by hand on the blackboard. They are not intended to explain or expound on any material.

## Bernoulli distribution

If $X$ encodes the outcome of a Bernoulli trial with success probability $p$ such that

$$X = \begin{cases} 1 & \text{if success} \\ 0 & \text{if failure,} \end{cases}$$

then $X$ has the *Bernoulli distribution* with success probability $p$.

We write $X \sim \text{Bernoulli}(p)$.

Derive pmf and cdf and compute mean and variance.

## Binomial distribution

If $X$ is the number of successes in $n$ independent Bernoulli trials, each with success probability $p$, then $X$ has the *Binomial distribution* with number of trials $n$ and success probability $p$.

We write $X \sim \text{Binomial}(n, p)$.

Derive pmf and cdf and compute mean and variance.

## Geometric distribution

If independent Bernoulli trials with success probability $p$ are conducted until the first success occurs and $X$ is the number of the trial on which the first success occurs, then $X$ has the *Geometric* distribution with success probability $p$.

We write $X \sim \text{Geometric}(p)$

Derive pmf and cdf and compute mean and variance.

## Negative binomial distribution

If independent Bernoulli trials with success probability $p$ are conducted until $r$ successes occur and $X$ is the number of the trial on which the $r$th success occurs, then $X$ has the *Negative binomial distribution* distribution with $r$ successes and success probability $p$.

We write $X \sim \text{NegativeBinomial}(p, r)$

Derive pmf and cdf and compute mean and variance.

## Hypergeometric distribution

Draw $K \geq 0$ marbles from a bag of $N \geq 0$ marbles, where $M \geq 0$ of the marbles are red. If $X$ is the number of red marbles drawn, then $X$ has the *Hypergeometric distribution*. We write $X \sim \text{Hypergeometric}(N, M, K)$.

Derive pmf and cdf and compute mean and variance.

## Discrete uniform distribution

If $X$ takes each of the values $1, \ldots, K$ with probability $1/K$ then $X$ has the *Discrete uniform distribution.* We write $X \sim \text{DiscreteUniform}(K)$.

Derive pmf and cdf and compute mean and variance.

## Empirical distribution of a set of points

Given a set of values $x_1, \ldots, x_n$, if $X$ takes each of the values $x_1, \ldots, x_n$ with probability $1/n$ then $X$ has the *Empirical distribution* of $x_1, \ldots, x_n$. We write

$X \sim \text{EmpiricalDistribution}(x_1, \ldots, x_n)$.

Derive pmf and cdf and compute mean and variance.

## Poisson distribution

The *Poisson distribution* is often posited for a random variable representing the number of occurrences of an event per unit of time/space, where the events

- occur independently from one another, and
- are as likely to occur in any time/space interval as in any other.

If on average $\lambda > 0$ events occur per unit of time/space, we might assume that $X$ has the Poisson distribution with mean $\lambda$.

We would write $X \sim \text{Poisson}(\lambda)$.

Give pmf and cdf and compute mean and variance.

# Relationships between some discrete distributions