STAT 511 fa 2019 Lec 09 slides

Quantiles

Karl B. Gregory

University of South Carolina

These slides are an instructional aid; their sole purpose is to display, during the lecture, definitions, plots, results, etc. which take too much time to write by hand on the blackboard. They are not intended to explain or expound on any material.

Quantiles

- Quantiles are percentiles, but not expressed as a percent; e.g. the 90th percentile is the 0.9 quantile.
- A baby whose head circumference is at the 0.9 quantile has a head circumference as great or greater than 90% of all babies.
- Random variables have quantiles. . .

The θth quantile of a continuous rv with strictly increasing cdf

Let X be a continuous rv with a strictly increasing cdf F_X. Then for any $\theta \in (0,1)$, the value q which satisfies

$$F_X(q) = \theta$$

is the *θ-quantile* of X.

So $X \leq q$ with probability θ.

Exercises: Find the 0.5, and 0.975 quantiles of

1. the distribution with cdf given by $F_Y(y) = e^{-e^{-y}}$ for all $y \in \mathbb{R}$.
2. the Normal$(0,1)$ distribution.

If F_X is not continuous and strictly increasing, then q which satisfies $F(q) = \theta$

- may not exist
- may not be unique

Example: How to find the $1/2$ quantile of $Y \sim \text{Binomial}(2, 1/2)$?

So we need a more general definition of quantiles.

Quantile function

The *quantile* function Q_X of a random variable X with cdf F_X is the function

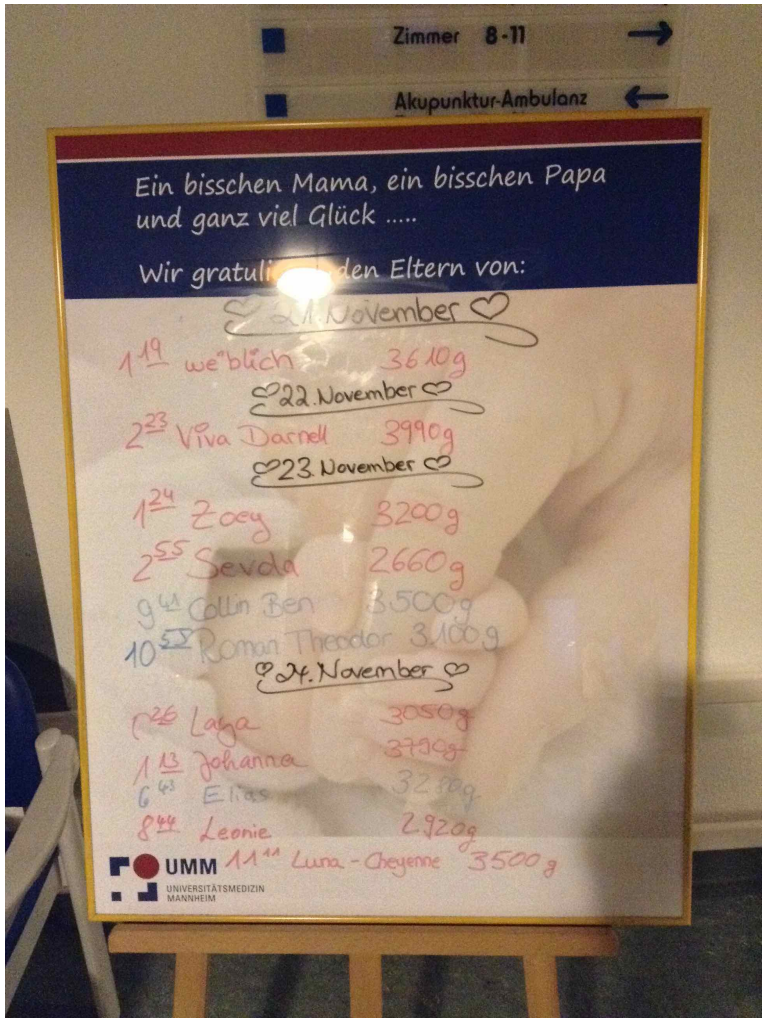
$$Q_X(\theta) = \inf\{x : F_X(x) \geq \theta\} \text{ for } \theta \in (0, 1).$$

The *θ-quantile* of the rv with cdf F_X is defined as $Q_X(\theta)$.

- If F_X is continuous and strictly increasing then $Q_X(\theta) = F_X^{-1}(\theta)$.
- $Q_X : (0, 1) \to \mathcal{X}$, where $\mathcal{X}$ the support of X.
- Q_X is always left-continuous.

Exercises: Find the quantile function of a rv with

1. the Binomial$(2, 1/2)$ distribution.
2. pdf given by $f_X(x) = x/2 \cdot \mathbf{1}(0 < x < 2)$.
3. the Exponential(λ) distribution.
4. the empirical distribution based the data points $x_1, \ldots, x_n$.



Name	Weight(g)
Sevda	2660
Leonie	2920
Laya	3050
Roman Theodor	3100
Zoey	3200
Elisa	3280
Collin Ben	3500
Luna-Cheyenne	3500
weiblich	3610
Johanna	3790
Viva Darnell	3990

Exercise: Find median, 0.25, and 0.90 quantiles of the empirical distribution.