STAT 511 fa 2019 Exam I

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- Do not open this test until told to do so.

- No calculators allowed; no notes allowed; no books allowed.

- Simplify all answers.

- SHOW YOUR WORK so that PARTIAL CREDIT may be given.

Chebychev's inequality: For any random variable X with mean μ_X and variance σ_X^2 and any constant $K > 0$, we have

$$P_X(|X - \mu_X| < K\sigma_X) \geq 1 - \frac{1}{K^2}.$$

1. Consider rolling a 6-sided die with sides ⊙, ⊡, ⊡, ⊡, ⊡, and ⊞ and define the random variable

$$X(s) = \begin{cases} 1 & \text{if } s \in \{⊙, ⊡, ⊡\} \\ 2 & \text{if } s \in \{⊡, ⊡\} \\ 3 & \text{if } s \in \{⊞\}. \end{cases}$$

(a) Give the support of X.

(b) Tabulate the probability distribution of X with a table of the form

| x	$\cdots$
$P_X(X = x)$	$\cdots$

(c) Write down the cdf F_X of X, making sure to define $F_X(x)$ for all $x \in \mathbb{R}$.

(d) Draw a detailed picture of the cdf F_X.

(e) Give the following probabilities:
   i. $P_X(X \le 1/2)$
   ii. $P_X(X \le 2.5)$
   iii. $P_X(1 < X \le 3)$

(f) Compute the expected value $\mathbb{E}X$ of X.

(g) Compute the variance $\operatorname{Var} X$ of X.

(h) Use Chebychev's inequality to give an interval within which X will fall with probability at least $1 - 1/16 = 0.9375$.

(i) Comment on whether you think the interval you gave in part (h) is useful for this random variable.

2. One of the two plots below shows the cdf of a random variable X and the other shows the pdf of the same random variable.
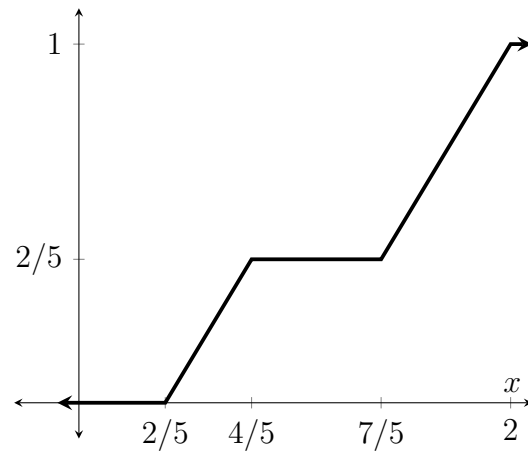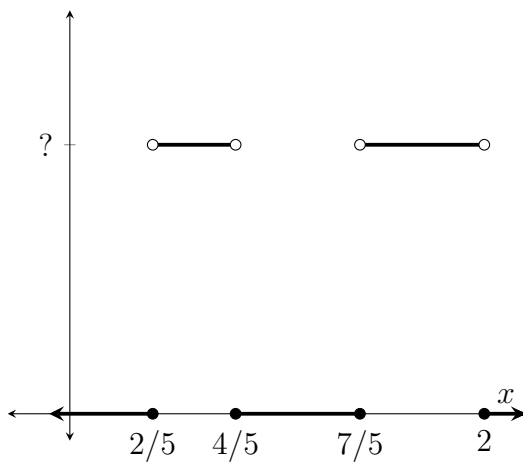
(a) Which plot shows the cdf?

(b) Is the random variable X discrete or continuous?

(c) Give the support of X.

(d) Give the following probabilities:

    i. $P_X(X \leq 1)$

    ii. $P_X(X = 7/5)$

    iii. $P_X(4/5 < X < 2)$

(e) Give the height of the function in the left-hand plot over the intervals $(2/5, 4/5)$ and $(7/5, 2)$.

3. (a) Give the number of unique sequences of letters that can be created with the letters in *borogoves*. You do not need to simplify your answer.

  (b) Consider the following set of words:

$$\textit{jaws} \quad \textit{the} \quad \textit{that} \quad \textit{claws} \quad \textit{the} \quad \textit{catch} \quad \textit{bite} \quad \textit{that}$$

    i. Suppose you draw two words without replacement from the above set of words. Give the probability that you draw the words *claws* and *the*. The order in which you draw them does not matter. Simplify your answer.

    ii. Suppose you draw one word at a time from the above set of words until you have drawn all the words. Give the probability that your sequence of draws results in the phrase *the jaws that bite the claws that catch*. You do not have to simplify your answer.

4. Suppose $1/10$ of all the text messages you receive come from family members, and $1/5$ of the messages from family members come before 8:00 am. In addition, suppose that $19/20$ of the messages you receive from non-family members come after 8:00 am.

(a) What is the proportion of text messages you receive before 8:00 am?

(b) If you receive a text message before 8:00 am, what is the probability that it is from a family member?