STAT 511 su 2020 hw 1

Basics of sets and probability

1. An experiment consists of flipping a coin and rolling a 6-sided die. Define the events

$$A = \text{“coin toss is a head”}$$
$$B = \text{“coin toss is a tail”}$$
$$C = \text{“odd number rolled”}$$
$$D = \text{“even number rolled”}$$
$$E = \text{“number greater than 3 rolled”}$$

   (a) List all the points the sample space $\mathcal{S}$ for the experiment.
   (b) List the sample points in the set $A \cup C$.
   (c) List the sample points in the set $A \cap C$.
   (d) List the sample points in E^C.
   (e) List the sample points in $A \cap C \cap E$.
   (f) List the sample points in $A \cap (C \cup E)$.
   (g) List the sample points in $(E \cap A) \cup (E \cap B)$.
   (h) List the sample points in $(E^C \cap C) \cup (E^C \cap D)$.
   (i) Give two pairs of events in which the events are disjoint.
   (j) Give two different partitions of $\mathcal{S}$ consisting of sets from among A, B, C, D, and E.
   (k) How can we represent the set $A \cap B$?

2. For two events A and B use our elementary set operations $\cup$, $\cap$, and the complement to give representations of the following:
   (a) Both A and B occur.
   (b) Neither A nor B occurs; give two representations of this.
   (c) At least one of the events A and B occurs.
   (d) At least one of the events A and B does not occur; give two representations of this.
   (e) One of the events A and B occurs but not the other.

3. For some events $A_1, A_2, \dots, A_n$, use $\bigcup_{i=1}^n$ and $\bigcap_{i=1}^n$ and the complement to give representations of the following:
   (a) None of the events $A_1, \dots, A_n$ occur.
   (b) All of the events $A_1, \dots, A_n$ occur.
   (c) At least one of the events $A_1, \dots, A_n$ occurs.
   (d) At least one of the events $A_1, \dots, A_n$ does not occur.

4. For some events $B_1, \dots, B_n$, interpret the following in words:
   (a) $\left(\bigcap_{i=1}^{n} B_i\right)^C$
   (b) $\left(\bigcup_{i=1}^{n} B_i\right)^C$
   (c) $\bigcup_{i=1}^{n} B_i$
   (d) $\bigcap_{i=1}^{n} B_i$

5. For two events A and B, draw Venn diagrams to illustrate the following identities:
   (a) $A = (A \cap B) \cup (A \cap B^C)$.
   (b) If $A \subset B$ then $A = A \cap B$.
   (c) If $A \subset B$ then $B = A \cup (B \setminus A)$.

6. Merit raises are to be given to two employees from among five candidate employees. Three of the employees are women and two are men. Define the following events:

$$A = \text{“at least one man is selected for a merit raise”}$$
$$B = \text{“one man and one woman are selected for a merit raise”}$$

   (a) List all the sample points in the sample space $\mathcal{S}$ of the experiment of selecting two employees to receive merit raises.
   Hint: denote the three women by W_1, W_2, and W_3 and the two men by M_1 and M_2.
   (b) Use operations on A and B to express the event that two women receive merit raises.
   (c) Use operations on A and B to express the event that two men receive merit raises.
   (d) Interpret in words the event $A^C \cup B$.
   (e) If the recipients of merit raises are randomly selected such that each outcome in $\mathcal{S}$ is equally likely, with what probability will one woman and one man receive the raise?

7. Assume that the distribution of blood types in the US is the following:

		antigens in RBCs			
		A	B	AB	O
Rh factor	−	0.063	0.015	0.006	0.066
	+	0.357	0.085	0.034	0.374

The following table indicates which recipient/donor combinations are possible:

		Donor							
		O−	O+	A−	A+	B−	B+	AB−	AB+
Recipient	O−	✓	.	.	.	.	.	.	.
	O+	✓	✓	.	.	.	.	.	.
	A−	✓	.	✓	.	.	.	.	.
	A+	✓	✓	✓	✓	.	.	.	.
	B−	✓	.	.	.	✓	.	.	.
	B+	✓	✓	.	.	✓	✓	.	.
	AB−	✓	.	✓	.	✓	.	✓	.
	AB+	✓	✓	✓	✓	✓	✓	✓	✓

(a) Find the probability that an individual randomly selected from the US population can donate blood to an $AB-$ recipient.

(b) Find the probability that an individual randomly selected from the US population can receive blood from an $AB-$ donor.

(c) Find the probability that an individual randomly selected from the US population is Rh+.

(d) Find the probability that an individual randomly selected from the US population is Rh+ or has the B antigen in his or her red blood cells.

8. Consider flipping a coin four times and recording the sequence of heads and tails.

   (a) Letting H denote "heads" and T denote "tails", list all elements in the sample space $\mathcal{S}$.

   (b) Assuming that each point in the sample space is equally likely, what is the probability of observing exactly three heads?

   (c) List the sample points in the event $A =$ "exactly two heads come up".

   (d) Give the probability of A^C.

   (e) Give the probability that at least two tails come up.

9. Use the Kolmogorov axioms to show that for any two events A and B

$$P(A) = P(A \cap B) + P(A \cap B^C).$$

10. Suppose Kobe Bryant could make free throws with a success rate of 83.69%.

    (a) Give a lower bound for the probability with which he would make 5 out of 5 free throws.

    (b) Give a lower bound for the probability with which he would make 10 out of 10 free-throws.

    (c) Comment on the usefulness of these bounds.