# STAT 511 su 2020 hw 2

*Counting problems*

*For some problems it will be useful to use the function* `choose()` *in R to compute large combinations. Type* `?choose` *to read the documentation. See also the function* `factorial()`.

1. A lift brings a skier from a lodge to the top of a mountain; from there, three slopes lead to a hut (hut 1) halfway down the mountain and two different slopes lead to a different hut (hut 2) halfway down the mountain. From each of these two huts there are four slopes leading back to the lodge.
   (a) In how many ways can the skier ski from the top of the mountain to the lodge?
   (b) If the skier chooses one of the several ways at random such that each way is equally probable, with what probability will she choose one which passes by hut 1?

2. A skier must wear gloves, a scarf, goggles, boots, a hat, and a jacket.
   (a) In how many different sequences can the skier don these articles?
   (b) If the skier chooses a sequence at random in which to don these articles, with what probability does she don her jacket before her gloves?

3. A group of 10 tired skiers is awarded 7 free cups of hot cocoa and 3 free beers.
   (a) In how many ways can the skiers distribute the free drinks such that each skier gets one drink?
   (b) Suppose you are one skier among the 10 skiers; in how many of the possible ways to distribute the drinks do you, yourself, receive a beer?
   (c) If one of the possible ways to distribute the drinks is chosen at random, with what probability will you receive a beer?
   (d) If 2 skiers among the 10 do not drink beer, in how many ways can the drinks be distributed?

4. Consider reordering the letters of "`bamboozle`".
   (a) Show that 90,720 unique sequences of letters are possible.
   (b) If one of the possible reorderings is chosen at random, what is the probability that it contains the string "`bamboo`"?
   (c) If one of the possible reorderings is chosen at random, what is the probability that it contains the string "`ooze`"?

5. In the game Heckmeck am Bratwurmeck, players begin each turn by rolling 8 dice. Each die is like an ordinary 6-sided die except that the "six" is replaced by the depiction of a smiling worm.
   (a) Find the probability of rolling 8 worms.
   (b) Find the number of ways to roll exactly 5 worms.
   *Hint: First you must choose which 5 from among the 8 dice to come up worms; then find the number of ways in which the remaining dice can come up* not *worms.*

(c) Show that the probability of rolling 5 worms can be expressed as

$$\binom{8}{5}\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^{8-5}$$

(d) Find the probability of rolling at least one worm.
*Hint: Try 1 minus the probability of the complement event.*

6. In the Midwestern card game of Euchre, four players play in teams of two, using a deck of 24 cards from which each player is dealt a hand of 5 cards. The 24-card deck is made by keeping only the 9s, the 10s, the face cards, and the aces.

(a) Find the probability that you are dealt the jack of hearts.

(b) Find the probability that you are dealt the jack of hearts and the jack of diamonds.

(c) Find the probability that you are dealt two jacks of the same color.

(d) Find the probability that any of the four players is dealt two jacks of the same color.

(e) Find the probability that you are dealt cards which are all hearts.

(f) Find the probability that you are dealt cards all of the same suit.

(g) Find the probability that a team possesses in the hands of the two players all the cards of one suit.

7. A curbside farmstand offers a basket of bounty (B.O.B.) consisting of 8 veggie items randomly selected from the farmer's inventory. Suppose you show up to the farmstand at the end of the day and the following items remain in the inventory:

4 squash, 5 bell peppers, 3 bulbs of garlic, 8 sweet potatoes.

(a) Find the probability that you get all of the bell peppers and all of the garlic bulbs in your B.O.B.

(b) Find the probability that you get 2 of each veggie in your B.O.B.

(c) Suppose you insist on getting all five bell peppers, allowing the farmer to choose your remaining 3 veggies from among the other items in the inventory. Find the probability that you get one of each of the remaining items in your B.O.B.

8. *The canonical birthday problem*: Consider selecting $n$ people at random, and assume that their birthdays are uniformly distributed (each day equally likely) over a 365-day year.

(a) Give an expression for the probability that no two people among the $n$ people share a birthday.

(b) Give the smallest size $n$ of the group for which the probability that there is at least one shared birthday is greater than 0.95.
*Hint: You will have to search for the answer trying different values of $n$.*