

# STAT 512 sp 2018 Exam I

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*Do not open this test until told to do so; no calculators allowed; no notes allowed; no books allowed; show your work so that partial credit may be given.*

$$f_Y(y) = \begin{cases} f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right| & \text{for } y \in \mathcal{Y} \\ 0 & \text{for } y \notin \mathcal{Y}. \end{cases}$$

$$f_{(Y_1, Y_2)}(y_1, y_2) = \begin{cases} f_{(X_1, X_2)}(g_1^{-1}(y_1, y_2), g_2^{-1}(y_1, y_2)) |J(y_1, y_2)| & \text{for } (y_1, y_2) \in \mathcal{Y} \\ 0 & \text{for } (y_1, y_2) \notin \mathcal{Y}. \end{cases}$$

pmf/pdf	$\mathcal{X}$	$M_X(t)$	EX	Var $X$
$p_X(x; p) = p^x(1-p)^{1-x},$	$x = 0, 1$	$pe^t + (1-p)$	$p$	$p(1-p)$
$p_X(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x},$	$x = 0, 1, \dots, n$	$[pe^t + (1-p)]^n$	$np$	$np(1-p)$
$p_X(x; p) = (1-p)^{x-1} p,$	$x = 1, 2, \dots$	$\frac{pe^t}{1-(1-p)e^t}$	$p^{-1}$	$(1-p)p^{-2}$
$p_X(x; p, r) = \binom{x-1}{r-1} (1-p)^{x-r} p^r,$	$x = r, r+1, \dots$	$\left[ \frac{pe^t}{1-(1-p)e^t} \right]^r$	$rp^{-1}$	$r(1-p)p^{-2}$
$p_X(x; \lambda) = e^{-\lambda} \lambda^x / x!$	$x = 0, 1, \dots$	$e^{\lambda(e^t-1)}$	$\lambda$	$\lambda$
$p_X(x; N, M, K) = \binom{M}{x} \binom{N-M}{K-x} / \binom{N}{K}$	$x = 0, 1, \dots, K$	¡complicadísimo!		$\frac{KM}{N} \frac{(N-K)(N-M)}{N(N-1)}$
$p_X(x; K) = \frac{1}{K}$	$x = 1, \dots, K$	$\frac{1}{K} \sum_{x=1}^K e^{tx}$	$\frac{K+1}{2}$	$\frac{(K+1)(K-1)}{12}$
$p_X(x; x_1, \dots, x_n) = \frac{1}{n}$	$x = x_1, \dots, x_n$	$\frac{1}{n} \sum_{i=1}^n e^{tx_i}$	$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$	$\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$
$f_X(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$-\infty < x < \infty$	$e^{\mu t + \sigma^2 t^2/2}$	$\mu$	$\sigma^2$
$f_X(x; \alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right)$	$0 < x < \infty$	$(1-\beta t)^{-\alpha}$	$\alpha\beta$	$\alpha\beta^2$
$f_X(x; \lambda) = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right)$	$0 < x < \infty$	$(1-\lambda t)^{-1}$	$\lambda$	$\lambda^2$
$f_X(x; \nu) = \frac{1}{\Gamma(\nu/2)2^{\nu/2}} x^{\nu/2-1} \exp\left(-\frac{x}{2}\right)$	$0 < x < \infty$	$(1-2t)^{-\nu/2}$	$\nu$	$2\nu$
$f_X(x; \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$	$0 < x < 1$	$1 + \sum_{k=1}^{\infty} \frac{t^k}{k!} \left( \prod_{r=0}^{k-1} \frac{\alpha+r}{\alpha+\beta+r} \right)$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

The table below gives some values of the function  $\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ :

$z$	1.282	1.645	1.96	2.576
$\Phi(z)$	0.9	0.95	0.975	0.995

1. Let  $X \sim f_X(x) = \frac{1}{2}(1+x)\mathbb{1}(-1 < x < 1)$  and let  $Y = |X|$ .

- (a) Give the support of  $Y$ .
- (b) Find  $P_Y(Y \leq 1/2)$ .
- (c) Give the cdf of  $Y$ .
- (d) Give the pdf of  $Y$ .

2. Let  $X$  have the pdf given by

$$f_X(x) = \begin{cases} \alpha c^\alpha x^{-(\alpha+1)} & x \geq c \\ 0 & x < c \end{cases}$$

for some constants  $\alpha > 0$ ,  $c > 0$ , and let  $Y = \log(X/c)$ .

- (a) Give the support of  $Y$ .
- (b) Find the inverse of the transformation  $Y = \log(X/c)$ .
- (c) Give the pdf of  $Y$ .
- (d) Give a transformation of  $X$  such that the resulting rv has the uniform distribution on  $(0, 1)$ .

3. Let  $X_1, \dots, X_{10}$  be independent random variables with pmf given by

$$p_X(x) = (1/4)^x (3/4)^{1-x} \mathbb{1}(x \in \{0, 1\}).$$

- (a) Give the mgf of the random variable  $Y = X_1 + \dots + X_{10}$ .
  - (b) Write down an expression for the probability  $P_Y(Y \geq 5)$ .
4. Let  $X_1 \sim \text{Beta}(1, 1)$  and  $X_2 \sim \text{Beta}(2, 1)$  and let  $X_1$  and  $X_2$  be independent. Define the pair of rvs  $(Y_1, Y_2)$  as  $Y_1 = X_1 X_2$  and  $Y_2 = X_2$ .
- (a) Write down the joint pdf of  $(X_1, X_2)$ .
  - (b) Give the support of  $(Y_1, Y_2)$ .
  - (c) Find the joint pdf of  $(Y_1, Y_2)$ .
  - (d) Find the marginal pdf of  $Y_1$ .