STAT 512 su 2021 Exam I

Karl B. Gregory

This is a take-home test. Do not communicate with classmates about the exam until after its due date/time. You may

- *Use your notes and the lecture notes.*
- *Use books.*
- *NOT work together with others.*

Write all answers on blank sheets of paper; then take pictures and merge to a PDF. Upload a single PDF to Blackboard.

1. Copy down this sentence on your answer sheet and put your signature underneath: *I have not collaborated with any other student on this exam. The work I have presented is my own.*

2. Let X be the sum of two rolls of a die and let Y be the remainder when 12 is divided by X.
   (a) Give the support $\mathcal{X}$ of X.
   (b) Give the support $\mathcal{Y}$ of Y.
   (c) Letting g represent the transformation from X to Y, give
      i. $g^{-1}(0)$
      ii. $g^{-1}(1)$.
   (d) Make a table giving the probability distribution of Y of the form

y	
$P(Y = y)$	

3. Let $X \sim \text{Beta}(\alpha, \beta)$ and let $Y = \tan(\pi(X - 1/2))$.
   (a) Give the support $\mathcal{X}$ of X.
   (b) Give the support $\mathcal{Y}$ of Y.
   (c) Give the pdf of Y.
   (d) Give the pdf of Y when $\alpha = \beta = 1$.

4. Consider the pdf $f_Y(y) = \pi^{-1}(1 + y^2)^{-1}$ for $y \in \mathbb{R}$.
   (a) Find the cdf F_Y corresponding to the pdf f_Y.
   (b) Find the transformation $g : (0, 1) \to \mathbb{R}$ such that the random variable $g(U)$ has density f_Y, where $U \sim \text{Uniform}(0, 1)$.
   (c) Let $Y_1, \ldots, Y_n$ be a random sample from the distribution with density f_Y. Find the pdf of the kth order statistic $Y_{(k)}$ of $Y_1, \ldots, Y_n$.

5. Let $X_1 \sim \text{Exponential}(\lambda_1)$ and $X_2 \sim \text{Exponential}(\lambda_2)$ be independent rvs for some $\lambda_1, \lambda_2 > 0$. Define new random variables $Y_1 = X_1/X_2$ and $Y_2 = X_2$.
   (a) Write down the joint pdf of X_1 and X_2.
   (b) Give the joint support of the rv pair (Y_1, Y_2).
   (c) Give the Jacobian of the transformation from (X_1, X_2) to (Y_1, Y_2).
   (d) Give the joint pdf of Y_1 and Y_2.
   (e) Find the cdf of Y_1 by evaluating $P(Y_1 \leq y_1) = P(X_1/X_2 \leq y_1)$ as a double integral.
   (f) Give the pdf of Y_1.

6. Let $Z_1, \ldots, Z_n \overset{\text{ind}}{\sim} \text{Normal}(0, 1/n)$ and let $h_1, \ldots, h_n \in \mathbb{R}$. Find the distribution of $S_n = \sum_{i=1}^n h_i \cdot Z_i$.

7. Let $X_1 \sim \text{Binomial}(n_1, p)$ and $X_2 \sim \text{Binomial}(n_2, p)$ be independent rvs. Let $Y = X_1 + X_2$. Give the cdf $F_Y(y)$ of Y.