STAT 513 hw 2

1. Suppose $X_1, \dots, X_n$ is a random sample from the Normal(μ, σ^2) distribution, where μ is unknown but σ^2 is known, and it is of interest to test H_0: $\mu = \mu_0$ versus H_1: $\mu \neq \mu_0$ for some value μ_0. The R code below plots the power curve of the test

$$\text{Reject } H_0 \text{ iff } |\sqrt{n}(\bar{X}_n - \mu_0)/\sigma| > z_{\alpha/2}$$

for user-selected values of μ_0, n, σ, and α. For a sequence of values of μ, the code computes the probability that the null hypothesis will be rejected according to the above test. In addition, for each value of μ in the sequence, a simulation is run: 100 data sets with sample size n are generated from the Normal(μ, σ^2) distribution, and for each of the 100 data sets, it is recorded whether the null hypothesis was rejected. For each value of μ, the proportion of times the null hypothesis is rejected is recorded. This gets plotted as a dashed line.

```
mu.0 <- ???
n <- ???
sigma <- ???
alpha <- ???

mu.seq <- seq(mu.0 - 5,mu.0 + 5,length=50)

z_crit <- qnorm(1-alpha/2)
power.theoretical <- 1-(pnorm(z_crit-sqrt(n)*(mu.seq - mu.0)/sigma)
                    -pnorm(-z_crit-sqrt(n)*(mu.seq - mu.0)/sigma))

power.empirical <- numeric()
for(j in 1:length(mu.seq))
{
    reject <- numeric()
    for(s in 1:100)
    {
                    x <- rnorm(n,mu.seq[j],sigma)
                    x.bar <- mean(x)

                    reject[s] <- abs(sqrt(n)*(x.bar-mu.0)/sigma) > z_crit
    }
    power.empirical[j] <- mean(reject)
}

plot(mu.seq,power.theoretical,type="l",ylim=c(0,1),xlab="mu",ylab="power")
lines(mu.seq, power.empirical,lty=2)
abline(v=mu.0,lty=3)  # vert line at null value
abline(h=alpha,lty=3) # horiz line at size
```

(a) Put in $\mu_0 = 2$, $n = 5$, $\sigma = 2$, and $\alpha = 0.05$ and execute the code. Turn in the plot.

(b) Explain why the dashed line follows the solid line closely but not exactly.

(c) Interpret the height of the solid line at $\mu = 4$.

(d) Interpret the height of the solid line at $\mu = 2$.

(e) Interpret the height of the dashed line at $\mu = 2$.

(f) What would be the effect on the height of the solid line at $\mu = 4$ if

i. the sample size n were increased?

ii. the standard deviation σ were increased?

iii. the size α of the test were increased?

(g) What would be the effect on the height of the solid line at $\mu = 2$ if

i. the sample size n were increased?

ii. the standard deviation σ were increased?

iii. the size α of the test were increased?

(h) What would be the effect on the dashed line of generating 500 data sets instead of only 100 data sets for the simulation at each value of μ?

2. Suppose $X_1, \dots, X_n$ is a random sample from the $\text{Normal}(\mu, \sigma^2)$ distribution, where μ and σ^2 are unknown, and it is of interest to test H_0: $\mu = \mu_0$ versus H_1: $\mu \neq \mu_0$ for some value μ_0. Consider the test

$$\text{Reject } H_0 \text{ iff } |\sqrt{n}(\bar{X}_n - \mu_0)/S_n| > t_{n-1,\alpha/2}.$$

(a) Modify the code in Question 1 so that it displays the true power curve and a simulated power curve of this test. Run your modified code with $\mu_0 = 2$, $n = 5$, $\sigma = 2$, and $\alpha = 0.05$. Turn in your code and the resulting plot. *Hint: Refer to Lec 02. You will need to specify a noncentrality parameter for the t distribution.*

(b) Compare the height of the solid curve at $\mu = 4$ with that of the solid curve in Question 1 at $\mu = 4$. Comment on whether there is a difference and why/why not.

(c) Compare the height of the solid curve at $\mu = 2$ with that of the solid curve in Question 1 at $\mu = 2$. Comment on whether there is a difference and why/why not.

3. Suppose the values

```
1.39 2.22 2.38 1.60 1.50
```

are a random sample from a distribution assumed to be Normal but for which the mean and variance are unknown.

(a) Give a 95% confidence interval for the mean.

(b) Test the hypotheses H_0: $\mu = 2.5$ versus H_1: $\mu \neq 2.5$ at the 0.05 significance level.

(c) Give the p-values for testing the following hypotheses:

i. H_0: $\mu = 2.5$ versus H_1: $\mu \neq 2.5$

ii. H_0: $\mu \leq 2.5$ versus H_1: $\mu > 2.5$

iii. H_0: $\mu \geq 2.5$ versus H_1: $\mu < 2.5$

iv. State whether a 99% confidence interval for μ based on this sample would contain the value 2.5.

4. Suppose the values

```
1.39 2.22 2.38 1.60 1.50
```

are a random sample from a distribution assumed to be Normal but for which the mean and variance are unknown.

(a) Report the estimated variance S_5^2.

(b) Give a 95% confidence interval for the unknown variance σ^2.

(c) Based on your answer to part (b), what is your conclusion about the hypotheses H_0: $\sigma^2 = 1.5$ versus H_1: $\sigma^2 \neq 1.5$ at the 0.05 significance level?

(d) Based on your answer to part (b), what is your conclusion about the hypotheses H_0: $\sigma^2 = 1.5$ versus H_1: $\sigma^2 \neq 1.5$ at the 0.01 significance level?

(e) Give the p-values for testing the following hypotheses:

i. H_0: $\sigma^2 \leq 1.5$ versus H_1: $\sigma^2 > 1.5$

ii. H_0: $\sigma^2 \geq 1.5$ versus H_1: $\sigma^2 < 1.5$

iii. H_0: $\sigma^2 = 1.5$ versus H_1: $\sigma^2 \neq 1.5$

5. You are importing a large number of small manufactured items and you want to know if more than 5% of them are defective. You randomly sample items one-by-one and determine whether each is defective or not. Let n be the number of items you sample and let Y be the number of defective items you discover out of the n items sampled. You decide to conclude that more that 5% are defective if $Y/n \geq 0.05 + 2\sqrt{(0.05)(0.95)/n}$.

(a) State the relevant null and alternate hypotheses.

(b) Suppose that the true proportion of defective items is 0.04 and that you discover two defective items out of $n = 10$ sampled items. Do you make a correct decision, a Type I error, or a Type II error?

(c) Suppose that the true proportion of defective items is 0.11 and you decide to sample $n = 10$ items. What is the correct conclusion, and what is the probability that you will come to the correct conclusion?

(d) If you sample $n = 100$ items and the true proportion of defective items is 0.05, what is the probability that you will make a Type I error?