

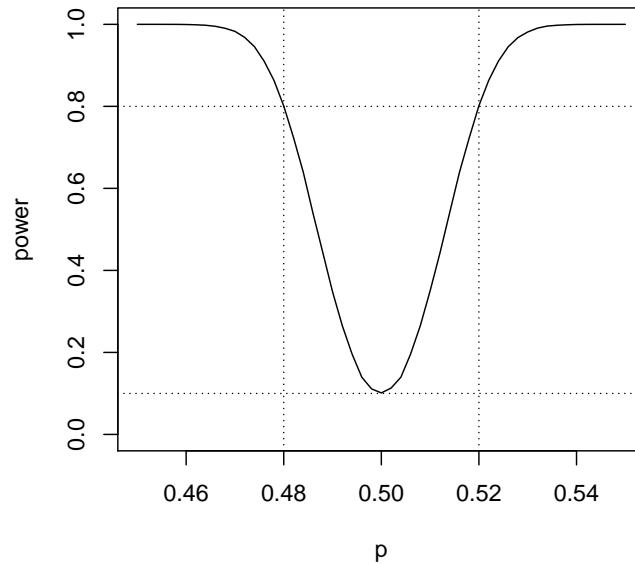
### STAT 513 hw 3

1. The following data resulted from an experiment studying the effect of nitrogen fertilizer on lettuce. Data are taken from Kuehl (2000).

Amount of fertilizer	Heads of lettuce per plot			
0 lbs/acre	104	114	90	140
200 lbs/acre	131	148	154	163

Assume that the number of heads of lettuce in the plots is approximately Normally distributed.

- (a) Let  $\sigma_1^2$  represent the variance of the number of heads of lettuce under 0 lbs/acre of fertilizer and let  $\sigma_2^2$  represent that under 200 lbs/acre. For each of the following sets of hypotheses get the  $p$ -value based on the data in the table.
- i.  $H_0: \sigma_1^2 \leq \sigma_2^2$  versus  $H_1: \sigma_1^2 > \sigma_2^2$
  - ii.  $H_0: \sigma_1^2 \geq \sigma_2^2$  versus  $H_1: \sigma_1^2 < \sigma_2^2$
  - iii.  $H_0: \sigma_1^2 = \sigma_2^2$  versus  $H_1: \sigma_1^2 \neq \sigma_2^2$
- (b) Do you believe that the variances  $\sigma_1^2$  and  $\sigma_2^2$  are different? Why/why not?
- (c) Let  $\mu_1$  represent the mean number of heads of lettuce per plot under 0 lbs/acre of fertilizer and let  $\mu_2$  represent that under 200 lbs/acre. For each of the following sets of hypotheses get the  $p$ -value based on the data in the table, assuming that  $\sigma_1^2 = \sigma_2^2$ .
- i.  $H_0: \mu_1 \leq \mu_2$  versus  $H_1: \mu_1 > \mu_2$
  - ii.  $H_0: \mu_1 \geq \mu_2$  versus  $H_1: \mu_1 < \mu_2$
  - iii.  $H_0: \mu_1 = \mu_2$  versus  $H_1: \mu_1 \neq \mu_2$
- (d) Do you believe the means  $\mu_1$  and  $\mu_2$  are different? Why/why not?
2. Suppose that in a poll of 743 registered voters, 350 say they will vote for candidate  $A$ . Suppose  $p$  is the proportion of all registered voters who will vote for candidate  $A$ .
- (a) Find the  $p$ -values for the following sets of hypotheses using the large-sample tests:
- i.  $H_0: p \leq 1/2$  versus  $H_1: p > 1/2$
  - ii.  $H_0: p \geq 1/2$  versus  $H_1: p < 1/2$
  - iii.  $H_0: p = 1/2$  versus  $H_1: p \neq 1/2$
- (b) Suppose a pollster wishes to detect a deviation in  $p$  from  $1/2$  as small as 0.03 in either direction with probability at least 0.90 while using a test with size  $\alpha = 0.05$ . What sample size should the pollster take?
- (c) For what sample size does the two-sided test have the power curve depicted below?  
*Hint: Identify  $\alpha$ ,  $\delta^*$ ,  $\gamma^*$ .*



3. Refer to the data in Question 1. Mr. Biogemüsebauer does not wish to fertilize his lettuce plots, but he wants to know if he can expect to get, without fertilizer, a mean yield of more than 100 heads of lettuce per plot (in plots of the same size as those used in the experiment). Otherwise, he will choose to use fertilizer. Let  $\mu$  be the mean number of heads of lettuce which grow per plot without fertilizer.
- Give the null and alternate hypotheses which are of interest to Mr. Biogemüsebauer.
  - Give the  $p$ -value for testing these hypotheses using the data from Question 1.
  - Suppose Mr. Biogemüsebauer wishes to conduct his own experiment using several lettuce plots. If the true mean lettuce yield without fertilizer is 110 heads per plot or more, he wishes to have a Type II error probability no greater than 0.10 when using a test with size 0.05. How many plots of lettuce should he use in his experiment? In your calculations, use the estimated variance for  $\sigma^2$ .

## References

Kuehl, R. O. (2000). *Designs of experiments: statistical principles of research design and analysis*. Duxbury Press.