

STAT 515 fa 2020 Exam I

Karl B. Gregory

This is a take-home test due to COVID-19. Do not communicate with classmates about the exam until after its due date/time. You may

- *Use your notes and the lecture notes.*
- *Use books.*
- *NOT work together with others.*

Write all answers on blank sheets of paper; then take pictures and merge to a PDF. Upload a single PDF to Blackboard.

1. Copy down this sentence on your answer sheet and put your signature underneath: *I have not collaborated with any other student on this exam. The work I have presented is my own.*
2. Suppose you have 5 vacancies in your symphonic wind ensemble: 2 trombonists, 2 trumpeters, and 1 oboist are needed. There are 11 applicants for the positions: 5 trombonists, 4 trumpeters, and 2 oboists.
 - (a) In how many ways could you select 2 from among the 5 trombonist applicants?
 - (b) In how many ways could the 5 vacancies be filled?
 - (c) You can conduct 3 auditions on Monday, 5 on Tuesday, and 3 on Wednesday. In how many ways can the applicants be assigned to the different days?
 - (d) Suppose you randomly select 3 of the applicants to be auditioned on Monday, and let X be the number of trombonists who are selected to be auditioned on Monday.
 - i. What is the support of the random variable X ?
 - ii. What is the name of the probability distribution of X ?
 - iii. Tabulate the probability distribution of X in a table like this:

x	\dots
$P(X = x)$	\dots

- iv. Give the expected value of X .
3. Suppose you are a vendor of chocolate covered donuts—with and without sprinkles. Suppose 40% of your customers are kids and that kids ask for sprinkles 90% of the time, while adults ask for no sprinkles 80% of the time. *Hint: Draw a tree diagram for this one.*
 - (a) Find the probability that a randomly selected customer is a kid and asks for sprinkles.
 - (b) Find the probability that a randomly selected customer asks for sprinkles.
 - (c) Find the probability that a randomly selected customer is a kid or asks for sprinkles.
 - (d) Find the probability that a randomly selected customer asks for no sprinkles.
 - (e) Find the probability that a randomly selected customer is an adult and asks for no sprinkles.
 - (f) Find the conditional probability that a customer is a kid given that the customer asks for sprinkles.
 - (g) Find the conditional probability that a customer is not a kid given that the customer asks for no sprinkles.
4. In the game *Heckmeck am Bratwurmeck*, players begin each turn by rolling 8 dice. Each die is like an ordinary 6-sided die except that the “six” is replaced by the depiction of a smiling worm. Consider rolling the 8 dice:
 - (a) If X is the number of worms you roll, what is the name of the probability distribution of the random variable X ?
 - (b) Give the probability of rolling no worms at all.
 - (c) Give the probability of rolling at least one worm.
 - (d) Give the probability that you roll exactly 4 worms.

- (e) Give the probability that you roll 5 or more worms.
5. Let X be the body temperature of a randomly selected healthy person and assume X is Normally distributed with mean $\mu = 98$ and variance $\sigma^2 = 1/9$.
- (a) Find $P(X > 98)$.
 - (b) Give $P(X = 98)$.
 - (c) Find $P(X > 98.5)$
 - (d) Find $P(97 < X < 99)$.
 - (e) Find the 40th percentile of X , that is the temperature such that X is less than or equal to it with probability 0.40.
 - (f) Find the median of X , that is, the temperature such that X is less than or equal to it with probability 0.50.