

# STAT 515 fa 2020 Exam II

Karl B. Gregory

*This is a take-home test due to COVID-19. Do not communicate with classmates about the exam until after its due date/time. You may*

- *Use your notes and the lecture notes.*
- *Use books.*
- *NOT work together with others.*

*Write all answers on blank sheets of paper; then take pictures and merge to a PDF. Upload a single PDF to Blackboard.*

1. Copy down this sentence on your answer sheet and put your signature underneath: *I have not collaborated with any other student on this exam. The work I have presented is my own.*
2. Suppose the number of birds that visit your bird feeder while you are eating breakfast each day follows a Poisson( $\lambda = 8$ ) distribution.
  - (a) Find the probabilities of the following events:
    - i. Exactly 8 birds visit the feeder during breakfast.
    - ii. At least one bird visits the feeder during breakfast.
    - iii. Fewer than 12 birds visit the feeder during breakfast.
  - (b) Let  $Y$  be the number of birds that visit your feeder while you are eating breakfast over the course of a given week. Find
    - i.  $\mathbb{E}Y$ .
    - ii.  $P(Y = 60)$ .
    - iii.  $P(Y > 50)$ .
3. Suppose the weights of peaches of a certain variety sold at a farmers market are Normally distributed with mean 147 grams and standard deviation 4 grams.
  - (a) Let  $X$  be the weight of a randomly selected peach.
    - i. Find  $P(X < 140)$ .
    - ii. Find  $P(|X - 147| < 3)$ .
    - iii.  $P(X = 147)$ .
    - iv. Find the weight exceeded by exactly 1% of the peaches of this variety.
  - (b) Let  $\bar{X}_n$  be the mean weight of  $n = 5$  randomly selected peaches.
    - i. Find  $P(\bar{X}_n < 144)$ .
    - ii. Find  $P(|\bar{X}_n - 147| < 3)$ .
    - iii. Find  $P(\bar{X}_n > 148)$ .
    - iv. Find  $P(\bar{X}_n = 147)$ .
    - v. Find the probability that the total weight of a bag of 5 peaches exceeds 740 grams.
4. A new variety of peaches has come to the farmers market. Suppose you buy a basket of 20 peaches having a total weight of 3,000 grams and that the standard deviation computed on the weights of the 20 peaches is  $S_n = 3.5$ . Assume that the peach weights are Normally distributed.
  - (a) Build a 95% confidence interval for the mean weight of peaches of this variety.
  - (b) Build a 99% confidence interval for the mean weight of peaches of this variety.
  - (c) Would a 90% confidence interval be wider or narrower than the 95% confidence interval?
  - (d) You would like to build a 95% confidence interval with a margin of error of no more than 1 gram. What sample size should you use?
  - (e) Suppose you want to test whether peaches of this new variety are heavier on average than peaches of the variety in question 3. Formulate the relevant hypotheses.

- (f) Based on your basket of 20 peaches, what is your conclusion about these hypotheses at the  $\alpha = 0.05$  significance level.
- (g) What is the  $p$ -value for testing these hypotheses based on your basket of 20 peaches?
- (h) Say whether you would reject the null hypothesis when testing

$$H_0: \mu = 147 \text{ versus } H_1: \mu \neq 147.$$

at the  $\alpha = 0.01$  significance level based on your basket of 20 peaches.

- (i) Make a 95% confidence interval for the variance  $\sigma^2$  of the weights of peaches of this new variety.

5. Suppose it is of interest to estimate the proportion  $p$  of students who eat a vegetarian diet.

- (a) Suppose you take a sample of 100 students and 5 say they are vegetarian.
  - i. Build a 95% confidence interval for  $p$ .
  - ii. If the true proportion of vegetarians is  $p = 0.034$ , give the exact value of  $P(\hat{p}_n \geq 0.05)$  for  $n = 100$ .
  - iii. Suppose you want to build a 95% confidence interval for  $p$  with a margin of error no greater than 1 percentage point. Using your sample of 100 students as a pilot study, suggest a sample size.
- (b) A food truck operator who serves vegetarian meals does his own study of the student population: in a random sample of 1000 students, he finds that 53 are vegetarian. He will visit campus with his food truck if he determines that the proportion of students who are vegetarian exceeds 4%.
  - i. State the hypotheses of interest to the food truck operator.
  - ii. State whether the food truck operator should reject or not reject his null hypothesis at significance level  $\alpha = 0.05$  based on the data he collected.
  - iii. Compute the  $p$ -value for testing his hypotheses based on the data he collected.