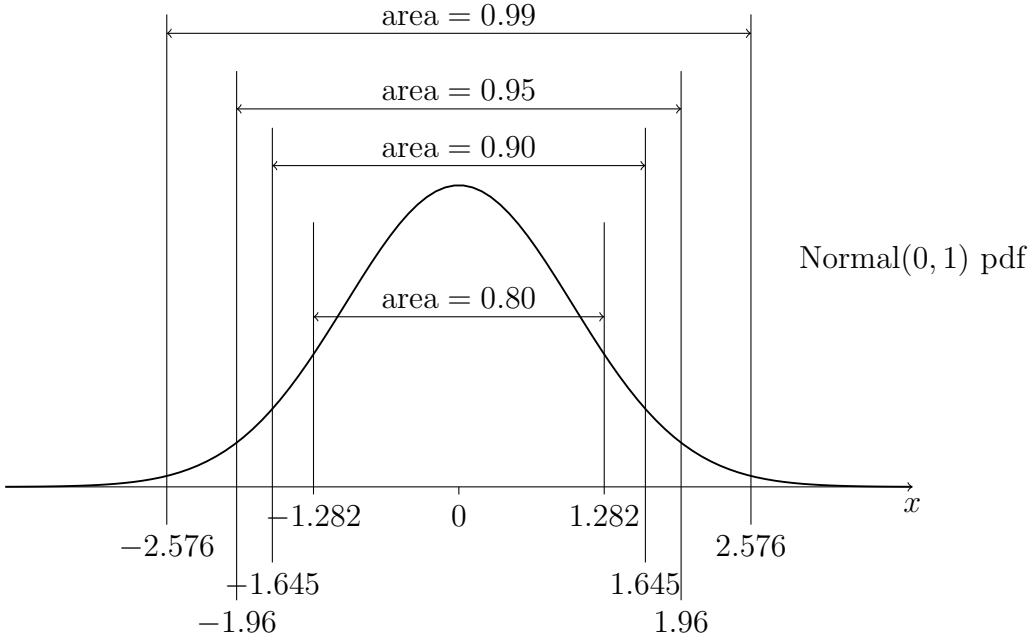
STAT 515 fa 2021 Exam I

Karl B. Gregory

- Do not open this exam until told to do so.
- You may have one handwritten sheet of notes out during the exam.
- You have 75 minutes to work on this exam.
- You may NOT use any kind of calculator.
- If you are unsure of what a question is asking for, do not hesitate to ask me for clarification.
- *Good luck, and may the odds be ever in your favor!*

$X \sim$		$\mathcal{X}$	$\mathbb{E}X$	$\text{Var}(X)$
Binomial(n,p)	$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$	$x = 0, 1, \ldots, n$	np	$np(1-p)$
Poisson(λ)	$P(X=x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$	$x = 0, 1, 2, \ldots$	λ	λ
Exponential(λ)	$P(X \leq x) = 1 - e^{-x\lambda}$	$x > 0$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$



1. Circle "True" or "False" for each of the following statements:
   (a) True False A larger sample size increases the variance of the sample mean.
   (b) True False Independence of two events implies that they are not mutually exclusive.
   (c) True False If X is the number of successes in a number of independent Bernoulli trials, then X has a hypergeometric distribution.
   (d) True False Every random variable has a probability density function.
   (e) True False The support of a random variable cannot contain negative numbers.

2. Suppose 5% of a population has a genetic mutation which causes individuals to develop, with probability 0.30, a type of cancer, whereas it is expected that 90% of individuals without the mutation will not develop the cancer.
   (a) What proportion of those with the cancer would you expect to possess the genetic mutation?
   (b) What proportion of individuals in the whole population would you expect to develop the cancer?

3. On average, 15 pilgrims per day visit a shrine, with all days and times of day being alike.
   (a) Suggest a probability distribution for the number of pilgrims visiting the shrine on a given day.
   (b) Suggest a probability distribution for the number of pilgrims visiting the shrine in a given week.
   (c) Give an expression for the probability that fewer than 30 pilgrims visit the shrine in the next two days (you do not need to evaluate the expression).
   (d) If each pilgrim, on arriving at the shrine, lights a candle which burns for one hour, with what probability will the candle be burning when the next pilgrim arrives?

4. A variety of asparagus produces spears with diameters having the Normal distribution with mean $\mu = 13$mm and standard deviation $\sigma = 1.5$mm. A packing house that prepares bundles of spears to be sold in supermarkets wants no more than 2% of the spears to have diameter less than 10mm.

   (a) Find the proportion of spears from this variety with diameter less than 10mm.

   (b) Give an interval, centered at the mean, within which 90% of the spear diameters lie.

   (c) Suppose the mean spear diameter can be increased by amending the soil. To what must μ be increased in order that no more than 2% of the spears will have diameters less than 10mm?

   (d) If the packing house samples 4 spears at random, with what probability will the mean of their diameters be less than 11mm? (Use $\mu = 13$mm).

5. A crate of asparagus arrives at a packing house. It contains 500 spears, 30 of which have diameter less than 10mm. The packing house quality control worker samples 20 spears. Give mathematical expressions for the following (you do not need to evaluate them):

   (a) If he samples with replacement,

      i. the probability that two or more of the sampled spears have diameter less than 10mm.

      ii. the probability that none of the sampled spears has diameter less than 10mm.

   (b) If he samples without replacement,

      i. the probability that two or more of the sampled spears have diameter less than 10mm.

      ii. the probability that none of the sampled spears has diameter less than 10mm.

   (c) Comment on whether the answers to parts (a) and (b) would be close to each other or not (if you were to compute them), and say why.