1. Assume the number of broken eggs in a carton of 18 eggs follows a binomial distribution, with the probability of any egg being broken equal to 1/50.

n

p

(a) Give the probability that no eggs in a carton are broken.

$$P(x=0) = \binom{18}{0}\left(\frac{1}{50}\right)^{0}\left(1-\frac{1}{50}\right)^{18-0} = \left(\frac{49}{50}\right)^{18}$$

(b) Give the probability that at least one egg in a carton is broken.

$$P(x\geq 1) = 1 - P(x=0)$$

$$= 1-\left(\frac{49}{50}\right)^{18}$$

(c) Give an expression for the probability that exactly 7 eggs in a carton are broken.

$$P(x=7) = \binom{18}{7}\left(\frac{1}{50}\right)^{7}\left(1-\frac{1}{50}\right)^{11}$$

18,000 eggs

(d) If you bought 1,000 cartons of eggs, recorded the number of broken eggs in each one, and then averaged these 1,000 numbers, to what value do you expect your average would be close?

- n·p

$$18\cdot\frac{1}{50} = \frac{18}{50} = \frac{9}{25}$$

Poisson(4)

2. Suppose the number of cyclists passing by an observation point on a road in one hour is a Poisson random variable such that on average 4 cyclists pass by in an hour.

(a) Give the probability that exactly 3 cyclists pass by in an hour (you do not have to simplify our expression to a number).

$P(x=3) = \boxed{\frac{4^3 e^{-4}}{3!}}$ ✓

(b) Give an expression for the probability that at least one cyclist passes by.

$1 - P(x=0) = 1 - \left[\frac{4^0 e^{-4}}{0!}\right] = \boxed{1 - e^{-4}}$ ✓

(c) Give the average time elapsed between the passing by of cyclists. = exponential dist

Exponential $E X = 1/\lambda = 1/4$

$\boxed{1/4 \text{ hour}}$ ✓

(d) Give the distribution of the number of cyclists passing by in two hours.

Poisson(λt) = Poisson$(4 \cdot 2)$ = Poisson(8) $= \frac{8^x e^{-8}}{x!}$ ✓

3. Two white wines and three red wines are to be blindly tasted. Your friend who considers himself a connoisseur will detest any white wine with probability 3/4 and detest any red wine with probability 1/4. He blindly selects a wine and tastes it... A = Detests wine B = White Wine

(a) Give the probability that he detests the randomly selected wine.

$P(A|B) = 0.75 \quad P(B) = 0.4$

$P(A|B^c) = 0.25 \quad P(B^c) = 0.6$

$P(A)$ Law of total Probability

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

$$= 0.75 \cdot 0.4 + 0.25 \cdot 0.6 \qquad 0.3 + 0.15 = \boxed{0.45}$$

(b) Suppose he detests the wine; with what probability has he selected a white wine?

Bayes

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{0.3}{0.45} = \frac{2}{3}$$

$\frac{3}{10} \cdot \frac{20}{9}$

$$\boxed{\frac{2}{3}}$$

(c) Suppose he likes the wine; with what probability has he selected a white wine?

$$P(B|A^c) = \frac{P(A^c|B) \cdot P(B)}{P(A^c)}$$

$P(A^c) = 1 - P(A) = 0.55$

$P(A^c|B) = 0.25 \quad P(B) = 0.4$

$$\boxed{= \frac{0.25 \cdot 0.4}{0.55}} = \frac{2}{11}$$

$\frac{1}{4} \cdot \frac{2}{5} \quad \frac{1}{10}$

$\frac{11}{20} \quad \frac{20}{11} \cdot \frac{1}{10}$

(d) Are the events that he selects a red wine and that he detests the selected wine independent?

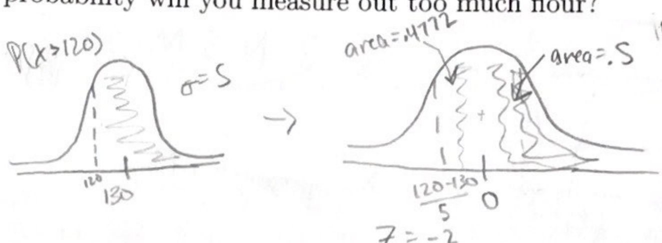
No they are not independent,

$$P(A) \neq P(A|B^c)$$

4. A European cookie recipe calls for 120 grams of flour. Suppose that when 1 cup of flour is measured out, the weight of the measured flour in grams follows a normal distribution with mean 130 and standard deviation 5. Lacking a kitchen scale, you measure the flour with your measuring cup and bake...

(a) With what probability will you measure out too much flour?

$X \sim Normal(\mu = 130, \sigma^2 = 5^2)$ $P(X > 120)$



$P(X > 120) = .4772 + .5$

$\boxed{= .9772}$ ✓

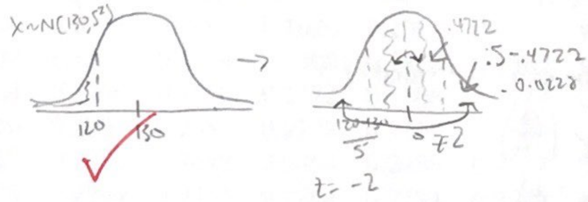
(b) With what probability will you measure out too little flour?

$P(X < 120) = 1 - P(X > 120)$

$= 1 - .9772$

$\boxed{= 0.0228}$ ✓

$X \sim N(130, 5^2)$



.49910
.5000
.4772
.0228

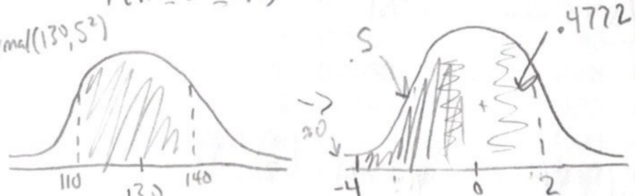
(c) With what probability will you measure out exactly 120 grams of flour?

$\boxed{P(X=120) = 0}$ ✓

(d) The cookies will turn out splendidly as long as the amount of flour used is between 110 and 140 grams. With what probability will the cookies turn out splendidly?

$P(110 \le x \le 140)$

$X \sim Normal(130, 5^2)$

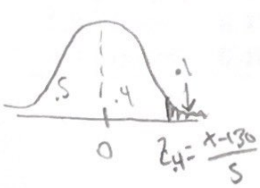


$P(110 < x < 140) = P(x > 110) + P(x < 140)$

$= .5 + .4772$

$\boxed{= .9772}$ ✓

(e) Give the amount of flour in grams such that only 10% of measured cups of flour contain a greater amount.



$z_{.4} = \frac{x - 130}{5}$

$x = 130 + 5(z_{.4})$

$x = 130 + 5(1.28)$

$\boxed{q_{.10} = 136.4}$ ✓

$\frac{8}{1} \cdot \frac{128}{100}$

6.4

$M=2$

$K=3$ choose 3   without replacement $N=5$

5. You may reach into a bag of candies and grab 3. The bag contains 5 candies, 2 of which are of your favorite kind. Let X be the number of your favorite kind of candy you grab.

   (a) Tabulate the probability distribution of X. Simplify all expressions.

x	0	1	2
$P(X=x)$	$\frac{\binom{2}{0}\binom{3}{3}}{\binom{5}{3}}$	$\frac{\binom{2}{1}\binom{3}{2}}{\binom{5}{3}}$	$\frac{\binom{2}{2}\binom{3}{1}}{\binom{5}{3}}$
(simplified) $P(X=x)$	$\frac{1}{10}$	$\frac{3}{5}$	$\frac{3}{10}$

$1\cdot2\cdot3\cdot4\cdot5$ 120

$\binom{2}{5}$ $\frac{5!}{3!\,2!}$ $\frac{120}{12}=10$

$\frac{6}{2\cdot1}=3$ 2

   (b) Give the expected value of X.

$\cancel{0\cdot\frac{1}{10}}+1\cdot\frac{3}{5}+2\cdot\frac{3}{10}$

$\frac{6}{1\cdot(2!)}=3$

$\frac{6}{10}+\frac{6}{10}=\frac{12}{10}=\frac{6}{5}$

$\frac{9}{25}\cdot\frac{3}{5}$ $\frac{27}{125}$

   (c) Now suppose that the bag contains a practically infinite number of candies, with two-fifths of the candies of your favorite kind. You may reach in and grab 3. Tabulate the distribution of X, the number of your favorite kind of candy you grab. Simplify all expressions.

$p=\frac{2}{5}$

0: XXX $\left(\frac{3}{5}\right)^3$

1: XXY, YXX, XYX $3\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^2$

2: XYY, YYX, YXY

YYY $\left(\frac{2}{5}\right)^3$

x	0	1	2	3
$P(X=x)$	$\left(\frac{3}{5}\right)^3$	$3\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^2$	$3\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)$	$\left(\frac{2}{5}\right)^3$
$P(X=x)$ simplified	$\frac{27}{125}$	$\frac{54}{125}$	$\frac{36}{125}$	$\frac{8}{125}$

$\frac{9}{25}\cdot\frac{6}{5}\cdot\frac{54}{125}$

$\frac{4}{25}$ $\frac{12}{25}\cdot\frac{3}{5}$ $\frac{36}{125}$

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