STAT 515 hw 1

Basics of sets, basic probability

1. Consider rolling two dice and let

$A =$ both rolls are at least 3
$B =$ both rolls are 3 or less
$C =$ the sum of the rolls is 10 or more
$D =$ the absolute value of the difference between the rolls is at most 1.

Give the following probabilities:

(a) $P(A)$

The possible outcomes are

$$\mathcal{S} = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\},$$

so we obtain $P(A) = 16/36$.

(b) $P(B)$

We have $P(B) = 9/36$.

(c) $P(C)$

We have $P(C) = 6/36$.

(d) $P(D)$

We have $P(D) = 16/36$.

(e) $P(A \cup B)$

We have $P(A \cup B) = 24/36 = 2/3$.

(f) $P(A \cap B)$

We have $P(A \cap B) = 1/36$.

(g) $P(A \cap B^c)$

We have $P(A \cap B^c) = 15/36$.

(h) $P((A \cap B)^c)$

We have $P((A \cap B)^c) = 35/36$.

(i) $P(A^c \cup B^c)$

We have $P(A^c \cup B^c) = 35/36$, by De Morgan's Laws.

(j) $P((A \cup B)^c)$

We have $P((A \cup B)^c) = 12/36$.

(k) $P(A^c \cap B^c)$

We have $P(A^c \cap B^c) = 12/36$, by De Morgan's Laws.

(l) $P(C \cap D)$

We have $P(C \cap D) = 4/36$.

(m) $P(C \cup D^c)$

We have $P(C \cap D^c) = 24/36 = 2/3$.

Hint: Begin by listing all possible outcomes of rolling two dice, i.e. the sample space.

2. Consider a bag of marbles, 19 of which are green, 25 of which are blue, and 6 of which are red. Moreover, suppose 9 of the green marbles are opaque, 5 of the blue marbles are opaque, and 3 of the red marbles are opaque, and the rest of the marbles are transparent.

   (a) Suppose you draw one marble from the bag. Give the probability that you draw

      i. a red marble.

      6/50

      ii. a transparent green marble.

      10/50

      iii. an opaque marble.

17/50

iv. a marble that is either blue or opaque or both.

37/50

(b) Suppose you remove all the opaque marbles from the bag and then draw one marble. Give the probability that you draw

i. a green marble.

10/33

ii. a red or a blue marble.

23/33

3. Suppose you draw 1 athlete at random from a group of 100 athletes such that: 30 swim; 44 run; 9 swim and run; 5 swim, bike, and run; 11 swim and bike; 10 bike and run but do not swim; and 35 only bike. Let S, B, and R denote the events that the athlete you draw swims, bikes, and runs, respectively. Give the following probabilities:

(a) $P(S \cup R)$

We have $P(S \cup R) = 30/100 + 44/100 - 9/100 = 65/100$.

(b) $P(S \cap R^c)$

We have $P(S \cap R^c) = 30/100 - 9/100 = 21/100$.

(c) $P(B)$

We have $P(B) = 35/100 + 10/100 + 5/100 + (11 - 5)/100 = 56/100$.

(d) $P(S \cup B)$

We have $P(S \cup B) = 30/100 + 56/100 - 11/100 = 75/100$.

(e) $P((S \cap R) \cap B^c)$

We have $P((S \cap R) \cap B^c) = 4/100 = 1/25$.

(f) $P(S^c \cup R^c)$

We have $P(S^c \cup R^c) = P((S \cap R)^c) = 1 - P(S \cap R) = 1 - 9/100 = 91/100$.

(g) $P((R \cap B) \cup (R \cap B^c))$

We have $P((R \cap B) \cup (R \cap B^c)) = P(R) = 44/100$.