

STAT 515 hw 2

Counting, conditional probability, Bayes' rule, independence

1. An iteration of the board game Candyland has the following cards: 6 each of red, purple, yellow, blue, orange, and green cards, and then 4 each of double red, double purple, double yellow, double blue, double orange, and double green cards. In addition, there is one lollipop, one peppermint, one peanut, and one ice cream card.
 - (a) If you draw 5 cards, one at a time, placing each card back into the deck and shuffling before the next draw, how many sequences of draws are possible?
 - (b) If you draw 5 cards, one at a time, without placing each card back before drawing the next, what is the probability that one of your 5 cards will be the ice cream cone?
2. Three guests to a tea party sit down in three chairs. Then the host re-arranges them according to a pre-made seating chart
 - (a) What is the probability that all of the guests may stay in their chairs?
 - (b) What is the probability that at least one guest may stay in her chair? *Hint: Just write down all the sample points. Don't try to use counting rules.*
3. A train to the frontier will have 4 passenger cars, 3 cattle cars, and 2 luggage cars.
 - (a) Bandits will enter two cars, selected at random. Give the probability that the bandits enter:
 - i. Two passenger cars.
 - ii. A luggage car and a passenger car.
 - iii. At least one cattle car.
 - (b) Suppose the train cars are put in random order. With what probability are all the passenger cars connected (with no non-passenger car between any two passenger cars)?
 - (c) There are 14 head of cattle to be transported in the three cattle cars.
 - i. In how many ways can 5, 5, and 4 head of cattle, respectively, be put into the three cattle cars?
 - ii. Of the 14 head of cattle, 3 are Jersey cows. If 5, 5, and 4 of the 14 head of cattle are put into the three cattle cars at random, with what probability will the Jersey cows all be placed in the same car?
 - (d) Mr. and Mrs. Wilkins and their two daughters and three sons are boarding one of the passenger cars on the next stop.
 - i. In how many different orders can the members of the Wilkins family enter the car?
 - ii. In how many of these orders does Mrs. Wilkins precede Mr. Wilkins?
 - iii. In how many of these orders does Mrs. Wilkins and her two daughters precede Mr. Wilkins his three sons?
 - (e) At each of the 10 stops along Lil' Jonnie's journey on the train, he will decide whether or not to scrawl his initials somewhere at the train station. At how many unique sets of stations can Lil' Jonnie scrawl his initials?

4. Suppose there are 5 bowling balls which are identical except that one is magical and delivers, no matter what, a strike with probability $3/4$. Suppose you get a strike 1 out of 4 times on average when using non-magical bowling balls. You select one of the 5 balls at random and send it down the lane...

- (a) Give the probability that you get a strike.
- (b) Given that you got a strike, what is the probability you chose the magic bowling ball?
- (c) Suppose you choose a ball and with the same ball you get two strikes in a row. What is the probability that you chose the magic ball?

5. Consider a bag of marbles, 19 of which are green, 25 of which are blue, and 6 of which are red. Moreover, suppose 9 of the green marbles are opaque, 5 of the blue marbles are opaque, and 3 of the red marbles are opaque, and the rest of the marbles are transparent. Suppose you draw one marble from the bag and let G , B , and R be the events that the marble is green, blue, and red, respectively, and let O be the event that it is opaque.

- (a) Find $P(R|O)$.
- (b) Find $P(R|O^c)$.
- (c) Find $P(R)$.
- (d) Check whether R and O are independent.
- (e) Find $P(B^c|O)$.
- (f) Find $P(O|G)$.
- (g) Find $P(B \cup G|O)$.

6. In all the state of the union (SOTU) addresses since that of President George Washington in the year 1790 to that of President Barack Obama in the year 2016, a total of 69,158 sentences were spoken. In all SOTU addresses following that of President Woodrow Wilson in 1920, a total of 32,752 sentences were spoken; among these, 15,934 had a length of 20 words or more. Among the sentences spoken prior to this, 27,639 had a length of 20 words or more.
See <https://doi.org/10.46430/phen0061> if you want to play with this kind of data.

- (a) If you select a sentence at random from the SOTU addresses given after 1920, with what probability does it have a length of at least 20 words?
- (b) If you select a sentence at random from the SOTU addresses given before or during 1920, with what probability does it have a length of at least 20 words?
- (c) If you select a sentence at random from all the SOTU addresses from 1790 to 2016, with what probability does it have a length of at least 20 words?
- (d) If you select a sentence at random from all the SOTU addresses from 1790 to 2016, what is the probability that it belongs to a SOTU address given after 1920, given that it has a length of at least 20 words?

7. Suppose you order a new pair of swimming goggles and that they will be manufactured with a defect making them leaky with probability $1/40$.

- (a) What is the probability of receiving a defective pair, and then again receiving a defective pair when you re-order the goggles?
- (b) What assumption did you make in order to compute your answer to part (a)?
- (c) What is the probability that you receive a defective pair and then a functioning pair when you re-order the goggles?
- (d) Give an expression for the probability that you receive K defective pairs of goggles, where $K \geq 1$.