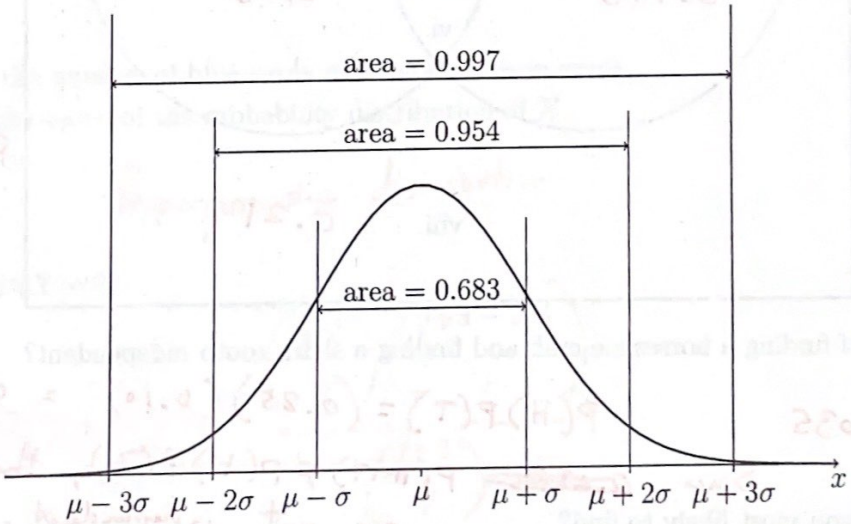
Solutions

STAT 515 sp 2026 Exam I

Karl Gregory

- Do not open this exam until told to do so.
- You may have one handwritten sheet of notes out during the exam.
- You have 75 minutes to work on this exam.
- You may NOT use any kind of calculator.

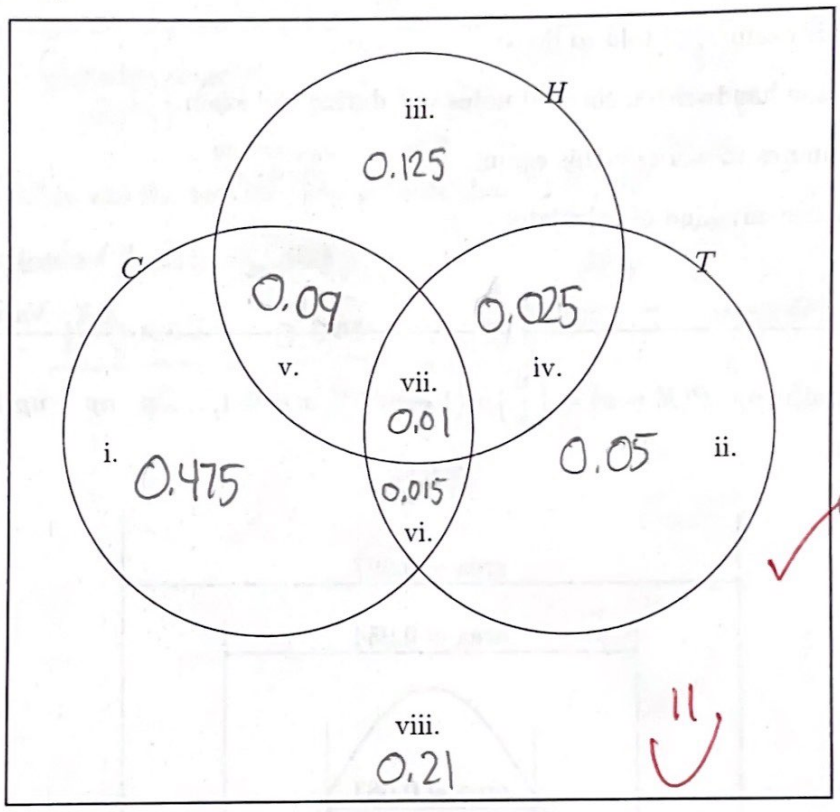
$X \sim$		$\mathcal{X}$	$\mathbb{E}X$	$\mathrm{Var}(X)$
Binomial(n,p)	$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$	$x=0,1,\ldots,n$	np	$np(1-p)$



1. You plan to make a list of items you find at the beach next time you go. Let H, C, and T denote the events that you find a horseshoe crab, a hermit crab, and a shark tooth, respectively, and suppose: The probability of finding only a horseshoe crab is 0.125; the probability of finding a shark tooth is 0.1; the probability of finding at least one of the items is 0.79; the probability of finding all three is 0.01; the probability of finding a horseshoe crab given that you find a shark tooth is 0.35; the probability you find a horseshoe crab and a hermit crab is 0.10, and the probability you find a hermit crab and a shark tooth is 0.025.

   (a) Give values for the probabilities in cells i.–viii. of the Venn diagram below.

P(Only H) = 0.125
P(T) = 0.1
P(At least one) = 0.79
P(None) = 1 − P(At least one)
= 1 − 0.79 = 0.21
P(H ∩ C ∩ T) = 0.01
P(H | T) = 0.35
P(H ∩ C) = 0.10
P(C ∩ T) = 0.025



v) 0.10 − 0.01 = 0.09
iv) $P(H|T) = \frac{P(H \cap T)}{P(T)}$
0.35 · 0.1 = P(H ∩ T)
P(H ∩ T) = 0.035
0.035 − 0.01 = 0.025
vi) P(C ∩ T) = 0.025,
0.025 − 0.01 = 0.015
ii) P(T) = 0.1,
0.1 − 0.025 − 0.01 − 0.0[illegible]
= 0.05
i) 0.79 − 0.1 − 0.125 − 0.0[illegible]
0.6 0.125 = 0.475

   (b) Are the events of finding a horseshoe crab and finding a shark tooth independent?

$P(H \cap T) = P(H)P(T)$
$0.035 = (0.25)(0.1) \rightarrow 0.035 \neq 0.025$
No, the events aren't independent

   (c) Which item are you most likely to find?

You are most likely to find a hermit crab.

H → (0.125 + 0.09 + 0.01 + 0.025) = 0.25
C → (0.475 + 0.09 + 0.01 + 0.015) = 0.59
T → (0.01 + 0.015 + 0.05 + 0.025) = 0.1

0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9
R, R, R, D, D, S, S, F, F, F, F, W, W, W, W

2. A standard UNO deck has 108 cards. In each of four colors there is a zero card, two cards of each number 1–9, and two each of *reverse*, *draw two*, and *skip* cards. In addition, there are four *draw four* cards and four *wild* cards not belonging to any color. Each player is dealt a hand of 7 cards from the deck.

   (a) Give the probability (you need not evaluate your expressions) of being dealt a hand composed of:

   i. Only yellow cards.

$$P(\text{only yellow}) = \frac{\binom{25}{7}}{\binom{108}{7}} \qquad \binom{N}{r} = \frac{N!}{r!(N-r)!}$$

yellow = 25

~~N = 108~~
~~n = 7~~
~~M = 25 yellow~~
~~x = yellow drawn~~

   ii. Cards all of the same color.

$$P(\text{cards all same color}) = \frac{4 \cdot \binom{25}{7}}{\binom{108}{7}}$$

each color has 25 cards
4 colors

   iii. Two *draw four* cards and five red cards.

$$\#\text{ways} = \binom{4}{2}\binom{25}{5} \qquad P(\text{2 draw 4 and 5 red}) = \frac{\binom{4}{2}\binom{25}{5}}{\binom{108}{7}}$$

   iv. At least one zero card.

$$P(\text{at least one zero}) = 1 - P(\text{no zero}) = 1 - \frac{\binom{104}{7}}{\binom{108}{7}}$$

zero cards = 4

   (b) Let X be the number of blue cards in a hand of seven cards.

X = # blue

   i. Give the name of the probability distribution of X.

$X \sim \text{hypergeometric}(N, M, n)$

Hypergeometric distribution

   ii. Give $P(X = 0)$.

$$P(X=x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{r}}$$

N = 108
n = 7
M = 25 blue cards
X = # blue drawn

$$P(X=0) = \frac{\binom{25}{0}\binom{108-25}{7-0}}{\binom{108}{7}}$$

$n = 20$

$P = 0.80$

3. Spudnik can throw his disc into a basket at a distance of 100 yards with probability of success 0.80. He will today make twenty attempts before a watching world. 20 trials

(a) Give an expression (you need not evaluate them) for the probability of the event that, in twenty attempts, from a distance of 100 yards, he throws his disc into the basket

i. zero times.

$$P(X=0) = \binom{20}{0}(0.8)^0(1-0.8)^{20-0}$$ ✓

ii. exactly ten times.

$$P(X=10) = \binom{20}{10}(0.8)^{10}(1-0.8)^{20-10}$$ ✓

iii. at least once.

$$P(X \geq 1) = 1 - P(X=0) = 1 - \left(\binom{20}{0}(0.8)^0(1-0.8)^{20-0}\right)$$ ✓

iv. five or more times.

$$P(X \geq 5) = \sum_{X=5}^{20} \binom{20}{x}(0.8)^x(1-0.8)^{20-x}$$ ✓

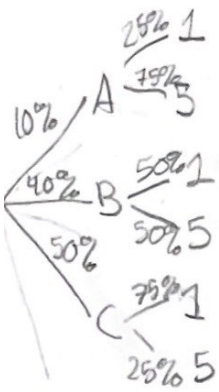
(b) What is the expected number of times Spudnik will throw his disc into the basket?

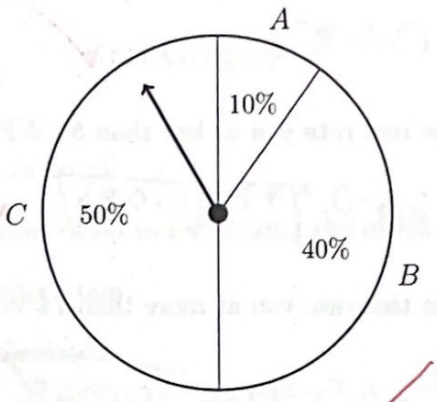
$EX = np \quad 20 \cdot .8 = 16$ ✓

(c) It has been noted of Spudnik that he becomes elated when his disc goes into the basket and despondent when it does not, and that he tends to follow up successful throws with successful throws and unsuccessful throws with unsuccessful throws. Given this information, what reason may there be to doubt the validity of the expressions given in part (a)?

The throws would no longer be independent of each other. Because of this the probability of success changes depending on the previous throw. ✓

4. The player of a game must spin a spinner like the one depicted below. The region in which the spinner stops spinning indicates from which of three bags the player will draw a single bill. Bag A contains one one-dollar bill and three five-dollar bills; bag B contains two one-dollar bills and two five-dollar bills; and bag C contains three one-dollar bills and one five-dollar bill.





(a) Give the probability of drawing a five-dollar bill.

$(0.1)(0.75) + (0.4)(0.5) + (0.5)(0.25)$
$= 0.075 + 0.2 + 0.125 = \boxed{0.4}$ or 40%

(b) Your buddy plays the game and draws a five-dollar bill; he says he will give it to you if you can guess which bag he drew from. Which bag will you guess? Justify your answer.

i will guess bag B because it has the greatest probability of getting \$5 by multiplying the chance of spinning B and the chance of getting \$5 from B

(c) Let X be the amount of money won from the game.

i. State whether X is a discrete or a continuous random variable.

This is a discrete variable as the range is not an interval

ii. Give the expected value of X. $EX = p_1x_1 + p_2x_2 +$

$EX = (0.1)(0.25)(1) + (0.1)(0.75)(5) + (0.4)(0.5)(1) + (0.4)(0.5)(5) + (0.5)(0.75)(1) +$
$= 0.025 + 0.375 + 0.2 + 1 + 0.375 + 0.625$ $(0.5)(0.25)($
$= \boxed{\$2.6}$

5. Suppose your typing speed, as judged by a particular typing test which provides a random selection of text, follows a normal distribution with a mean of 62 words per minute (WPM) and a standard deviation of 4 WPM.

   (a) With what probability will the test rate you between 58 and 66 WPM? 1σ

   0.683

   (b) With what probability will the test rate you at less than 54 WPM? 2σ

   $\frac{1-0.954}{2} = \frac{0.046}{2} = 0.023$

   (c) With what probability will the test rate you at more than 74 WPM? 3σ

   $\frac{1-0.997}{2} = \frac{0.003}{2} = 0.0015$