STAT 515 sp 2026 Exam II

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- Do not open this exam until told to do so.
- You may have two handwritten sheets of notes out during the exam.
- You have 75 minutes to work on this exam.
- You may NOT use any kind of calculator.

$X \sim$		$\mathcal{X}$	$\mathbb{E}X$	$\mathrm{Var}(X)$
Poisson(λ)	$P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$	$x = 0, 1, 2, \dots$	λ	λ
Exponential(λ)	$P(X \leq x) = 1 - e^{-x\lambda}$	$x > 0$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$

$$\hat{p}_n \pm z_{\alpha/2} \cdot \sqrt{\hat{p}_n(1 - \hat{p}_n)/n} \qquad \bar{X}_n \pm t_{n-1,\alpha/2} \cdot S_n/\sqrt{n}$$

$$Z_{\text{test}} = \frac{\hat{p}_n - p_0}{\sqrt{p_0(1 - p_0)/n}} \qquad T_{\text{test}} = \frac{\bar{X}_n - \mu_0}{S_n/\sqrt{n}}$$

A z-table and a t-table are appended to this exam.

1. A type of mushroom pops up in a hermit's forest such that when he walks through his forest he encounters, on average, 5 mushrooms every 100 yards. Suppose the mushrooms pop up in the forest according to a Poisson process. Here comes the hermit now…

   (a) Give the probability that he will walk 100 yards without encountering a single mushroom.

   (b) Give the probability that he encounters at least one mushroom in the first 100 yards.

   (c) Give an expression for the probability that he finds at least 5 mushrooms in the first 100 yards. You do not need to evaluate your expression.

   (d) Give the expected number of mushrooms he will encounter if he walks 300 yards in his forest.

   (e) Suppose he measures the distance from the first mushroom he encounters to the next mushroom he encounters. What probability distribution does this random variable have?

2. An espresso machine yields double shots with volumes following a normal distribution with mean 60 ml and standard deviation 2 ml.

   (a) Give the probability that the next double shot will have volume less than 59 ml.

   (b) Give the probability that the next double shot will have volume greater than 59 ml.

   (c) Give the 93.32th percentile of the distribution of volumes of double shots of espresso from this machine.

   (d) Suppose you pull 25 double shots of espresso. Give the probability that the average of the volumes of the 25 double shots is less than 59 ml.

   (e) Suppose the volumes were *not* normally distributed. What is required for the mean volumes of samples of double shots to have approximately a normal distribution?

3. A botanist sows 100 seeds of a certain type in a greenhouse and is interested in the probability that a seed of this type will germinate under the greenhouse conditions. After some time, she observes that 81 of the 100 seeds germinated.

   (a) Give an expression for a 99% confidence interval for the probability that this type of seed will germinate under the greenhouse conditions. You do not need to evaluate the bounds of the interval.

   (b) The botanist wishes to know if the probability of germination under her greenhouse conditions is more than 0.75. Formulate a null and an alternate hypothesis which correspond to her research question.

   (c) Write down an expression for the test statistic for testing the hypotheses in the previous part. You do not need to evaluate your expression.

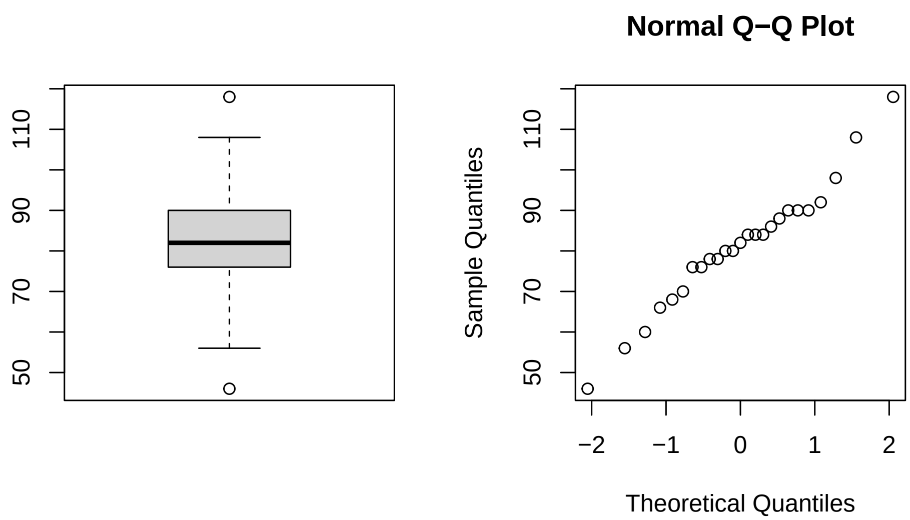
   (d) The test statistic value is 1.386. Does the researcher reject H_0 at the $\alpha = 0.01$ significance level?

   (e) Suppose the botanist plans to do a larger study; she wishes in the end to be able to construct a 99% confidence interval with margin of error no greater than two percentage points for the true probability that a seed of this type will germinate under her greenhouse conditions. Give an expression for a guess of the sample size needed for her to achieve this. You do not need to evaluate your expression.

4. Students in a class were asked to measure their heart rates, resulting in the following set of numbers.

80	84	108	70	84
118	76	90	68	80
92	82	90	46	78
84	76	90	98	66
78	88	86	56	60

The average of all the measurements is 81.12 and the standard deviation is 15.45. A boxplot and a Normal quantile-quantile plot are shown.



(a) Give an expression for a 95% confidence interval for the mean heart rate of this population. You do not have to evaluate the endpoints of your interval.

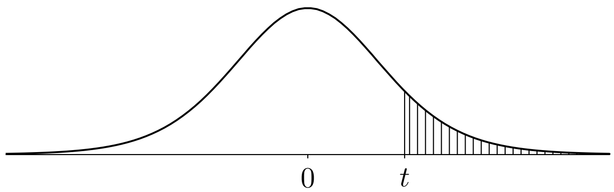
(b) Suppose it is of interest to know whether the mean heart rate in this population differs from 80 bpm. Write down the relevant null and alternate hypotheses.

(c) Give an expression for the test statistic for testing the hypotheses in the previous part. You do not have to evaluate your expression.

(d) The value of the test statistic is 0.362. State your decision about the hypotheses in part (b) at the $\alpha = 0.05$ significance level.

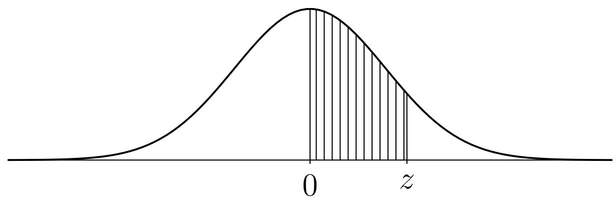
(e) Explain the purpose of the Normal quantile-quantile plot.

Upper tail probabilities of t-distributions



ν	α = 0.100	0.050	0.025	0.010	0.005	0.001	0.0005
1	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088	636.6192
2	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271	31.5991
3	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145	12.9240
4	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732	8.6103
5	1.4759	2.0150	2.5706	3.3649	4.0321	5.8934	6.8688
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076	5.9588
7	1.4149	1.8946	2.3646	2.9980	3.4995	4.7853	5.4079
8	1.3968	1.8595	2.3060	2.8965	3.3554	4.5008	5.0413
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.2968	4.7809
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437	4.5869
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.0247	4.4370
12	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296	4.3178
13	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520	4.2208
14	1.3450	1.7613	2.1448	2.6245	2.9768	3.7874	4.1405
15	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328	4.0728
16	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862	4.0150
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458	3.9651
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105	3.9216
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.5794	3.8834
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.5518	3.8495
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272	3.8193
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050	3.7921
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850	3.7676
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668	3.7454
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502	3.7251
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350	3.7066
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210	3.6896
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082	3.6739
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962	3.6594
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852	3.6460
31	1.3095	1.6955	2.0395	2.4528	2.7440	3.3749	3.6335
32	1.3086	1.6939	2.0369	2.4487	2.7385	3.3653	3.6218
33	1.3077	1.6924	2.0345	2.4448	2.7333	3.3563	3.6109
34	1.3070	1.6909	2.0322	2.4411	2.7284	3.3479	3.6007
35	1.3062	1.6896	2.0301	2.4377	2.7238	3.3400	3.5911
40	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069	3.5510
60	1.2958	1.6706	2.0003	2.3901	2.6603	3.2317	3.4602
120	1.2886	1.6577	1.9799	2.3578	2.6174	3.1595	3.3735
∞	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905

Standard Normal probabilities:



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990