

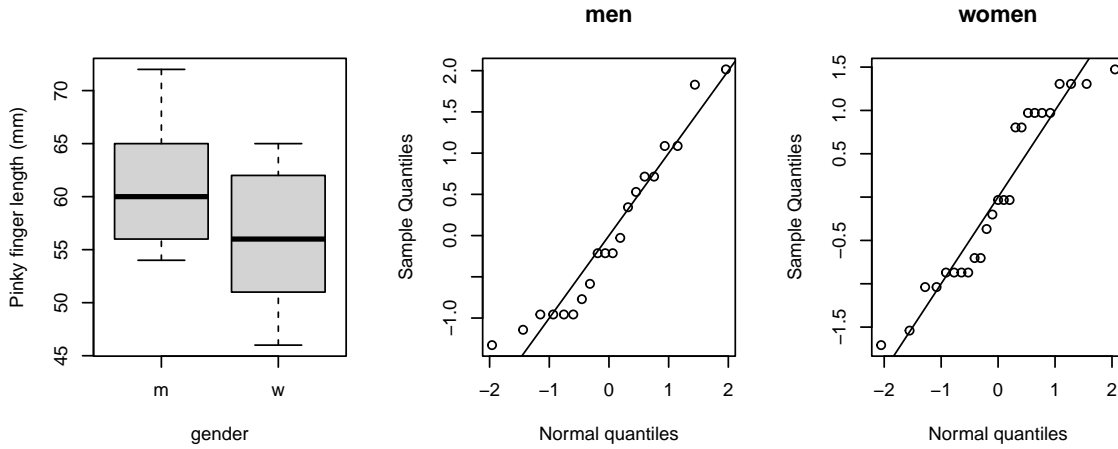
STAT 515 sp 2026 Final Exam

Karl Gregory

- Do not open this exam until told to do so.
- You may have three handwritten sheets of notes out during the exam.
- You have 150 minutes to work on this exam.
- You may NOT use any kind of calculator.

A z -table is appended to this exam.

1. The pinky finger lengths and genders of several statistics students were recorded. Below are some plots of the data and some R output.



Two Sample t-test

```
data:  pnk by gender
t = 2.885, df = 43, p-value = 0.006096
alternative hypothesis: true difference in means
between group m and group w is not equal to 0
95 percent confidence interval:
 1.489776 8.410224
sample estimates:
mean in group m mean in group w
      61.15      56.20
```

- (a) Explain the purpose of looking at the two plots on the right.

- (b) Write down the null and alternate hypotheses one should test in order to see if the average pinky finger length of men exceeds that of women.
- (c) Based on the R output, give the value of the test statistic for testing the hypotheses from part (b).
- (d) Give the p value for testing the hypotheses from part (b).
- (e) Based on the R output, what is your conclusion about the mean pinky finger lengths for men and women?

2. Suppose the orders of one hundred diners are recorded. Sixty of them order a salad, and, among these sixty, twenty order a dessert. Among the forty who do not order a salad, the number who order a dessert is also twenty.

(a) Summarize the counts in a two-by-two table. Include row and column sums in the margins.

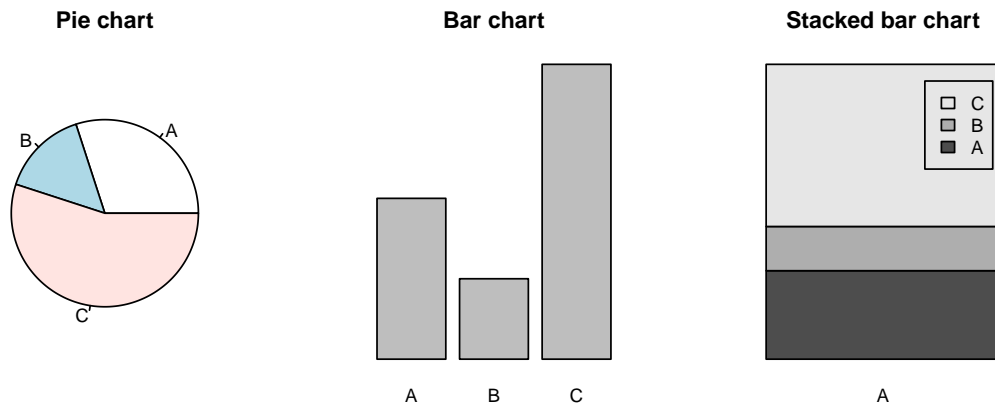
(b) It is of interest to test whether there is an association between ordering a salad and ordering a dessert. Write down the corresponding null and alternate hypotheses.

(c) Give the table of expected counts under the null hypothesis of no association.

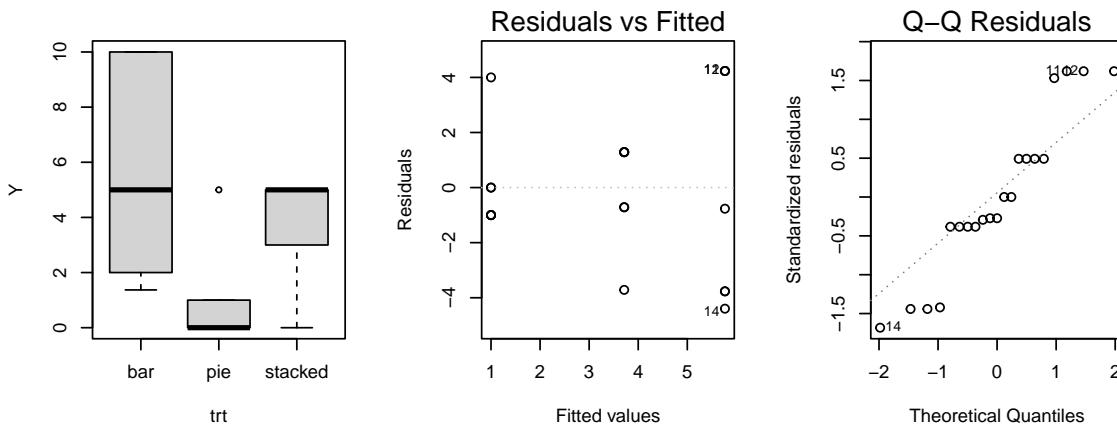
(d) Is the rule of thumb satisfied for using the chi-squared test of association? State the rule and explain why it is or is not satisfied.

(e) Pearson's test statistic for comparing the observed to the expected counts has the value 2.78. Carefully explain how we come to a decision whether to reject or not to reject the null hypothesis based on the value of the test statistic.

3. Each student in a statistics class was shown one of the three charts below and allowed 5 seconds to guess the percentage of the whole made up by category B:



The absolute values of the differences between the students' guesses and the true percentage made up by category B (which was 15%) were recorded as response values in a one-way ANOVA model. Some plots and output are shown below:



Analysis of Variance Table

Response: Y

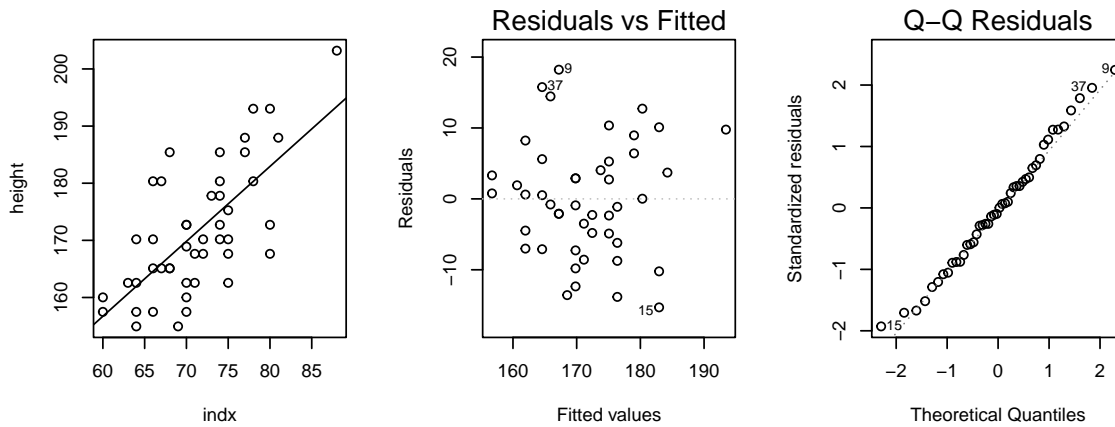
	Df	Sum Sq	Mean Sq	F value	Pr(>F)
trt	?	80.05	40.025	5.0211	0.0185 *
Residuals	18	143.49	7.971		

- Replace the question mark in the ANOVA table with the correct degrees of freedom value.
 - Give the null and alternate hypotheses for which the F value in the ANOVA table is the test statistic.
-
- State whether you believe the assumptions of the one-way ANOVA model to be satisfied. Explain your answer.

(d) How does one obtain the value 5.0211 from the other values in the ANOVA table?

(e) State your conclusions about the hypotheses in part (b) at the $\alpha = 0.05$ significance level.

4. Several statistics students recorded their heights (cm) and the lengths of their index fingers (mm). It is of interest to see whether height is linearly related to index finger length. Some plots and output are below.



Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	78.0144	14.7773	5.279	3.81e-06 ***
indx	1.3116	0.2067	6.346	1.04e-07 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.228 on 44 degrees of freedom

Multiple R-squared: 0.4779, Adjusted R-squared: 0.466

F-statistic: 40.27 on 1 and 44 DF, p-value: 1.045e-07

	2.5 %	97.5 %
(Intercept)	48.2327000	107.796061
indx	0.8950504	1.728079

- (a) What proportion of variability in height can be explained by considering index finger length?
- (b) State whether you believe the assumptions of the simple linear regression model are satisfied. Explain your answer.
- (c) Suppose you find the print of a hand with index finger length equal to 76 mm. Give an expression (you do not have to evaluate it) for your best guess of the height of the person who made the hand print.
- (d) What if you find the print of a hand with index finger length equal to 50 mm?

(e) State the null and alternate hypotheses for which the number 1.04×10^{-7} (which appears in the output) is the p value.

(f) Give an interpretation of the confidence interval $[0.895, 1.728]$ which appears in the output (I have rounded the numbers).

(g) Draw a dark circle on the scatterplot at your best guess of the location (\bar{x}_n, \bar{Y}_n) , where \bar{x}_n is the mean of the observed index finger lengths and \bar{Y}_n is the mean of the observed heights.

(h) Draw a small box around the point in the scatterplot which you believe has the highest leverage value.

5. Suppose there are 5 bowling balls which are identical except that one is magical and delivers, no matter what, a strike with probability $3/4$. Suppose you get a strike 1 out of 4 times on average when using non-magical bowling balls. You select one of the 5 balls at random and send it down the lane. . .

(a) Give the probability that you get a strike.

(b) Given that you got a strike, what is the probability you chose the magic bowling ball?

(c) Suppose you choose a ball and with the same ball you get two strikes in a row. What is the probability that you chose the magic ball?

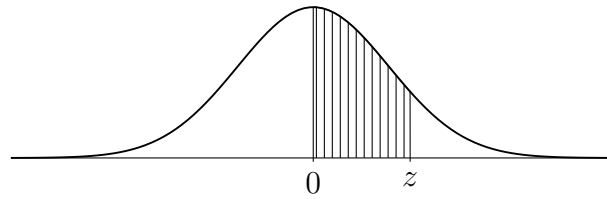
6. Suppose the index finger lengths of women are normally distributed with mean 56 mm and standard deviation 5 mm.

(a) Give the probability that the index finger length of a randomly selected woman exceeds 63.5 mm.

(b) A woman claims that the length of her index finger is at the 2.28 percentile of index finger lengths. What is the length of her index finger?

(c) Give the probability that the mean of the index finger lengths of sixteen randomly selected women will be less than 54.75 mm.

Standard Normal probabilities:



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990