STAT 516 Lec 01

Inference on the mean and variance of a Normal population

Confidence Intervals

Testing hypothesis

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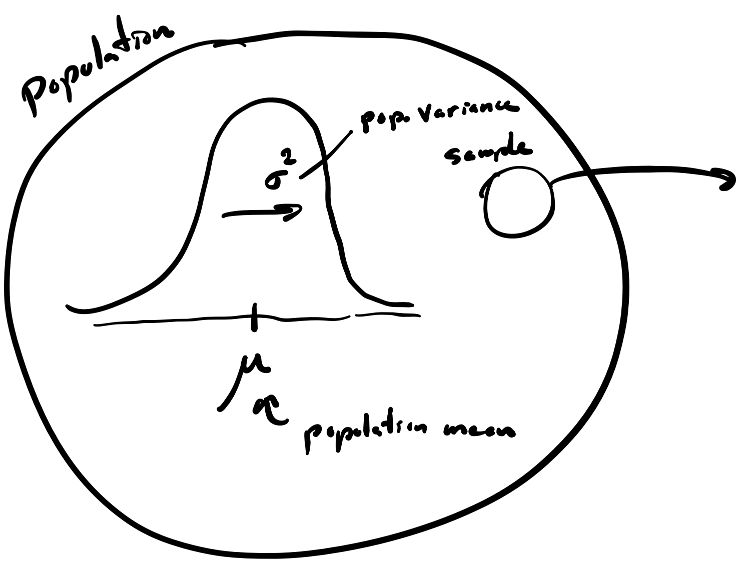


$\bar{X}_n$, S^2

sample mean

Sample variance

Setup

Throughout let $X_1, \dots, X_n \overset{\text{ind}}{\sim}$ Normal(μ, σ^2).

In this lecture we review how to:

1. Estimate μ and σ^2.
2. Build confidence intervals for μ and σ^2.
3. Test hypotheses concerning μ and σ^2.
4. Choose the sample size.

We call $X_1, \dots, X_n$ a random sample.

Golden ratio example:

95% C.I for μ: $[1.506, 1.623]$

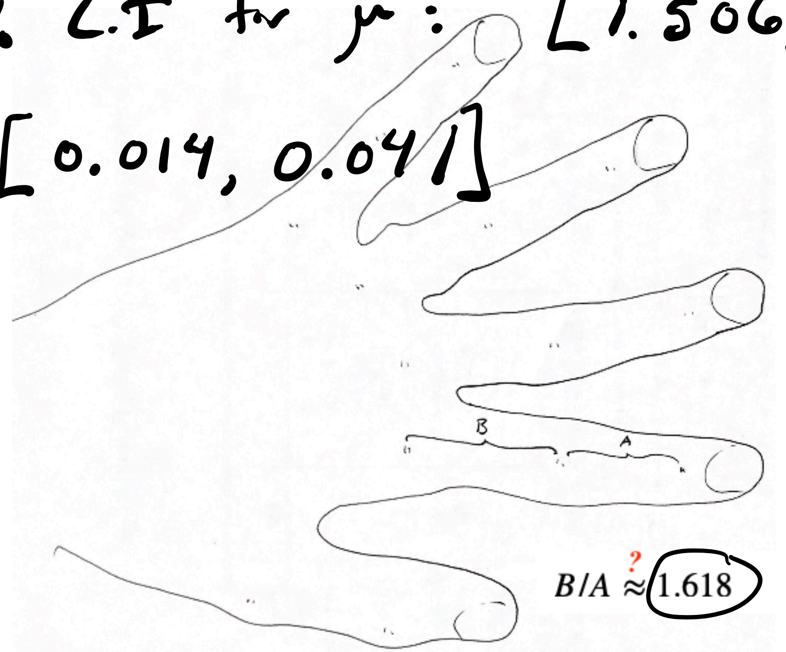
95% C.I for σ^2: $[0.014, 0.041]$

$\bar{X}_n = 1.56$

$S_n = 0.148$

C.I.:

$\bar{X}_n \pm$ Margin of error



$\frac{1+\sqrt{5}}{2}$

A class of $n = 27$ students measured B/A on themselves:

$H_0: \mu = 1.618$

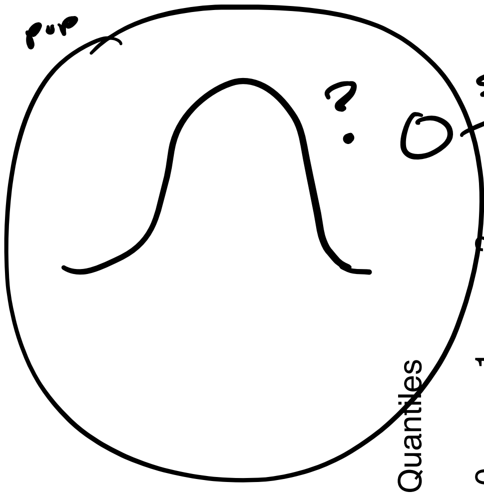
$H_1: \mu \neq 1.618$

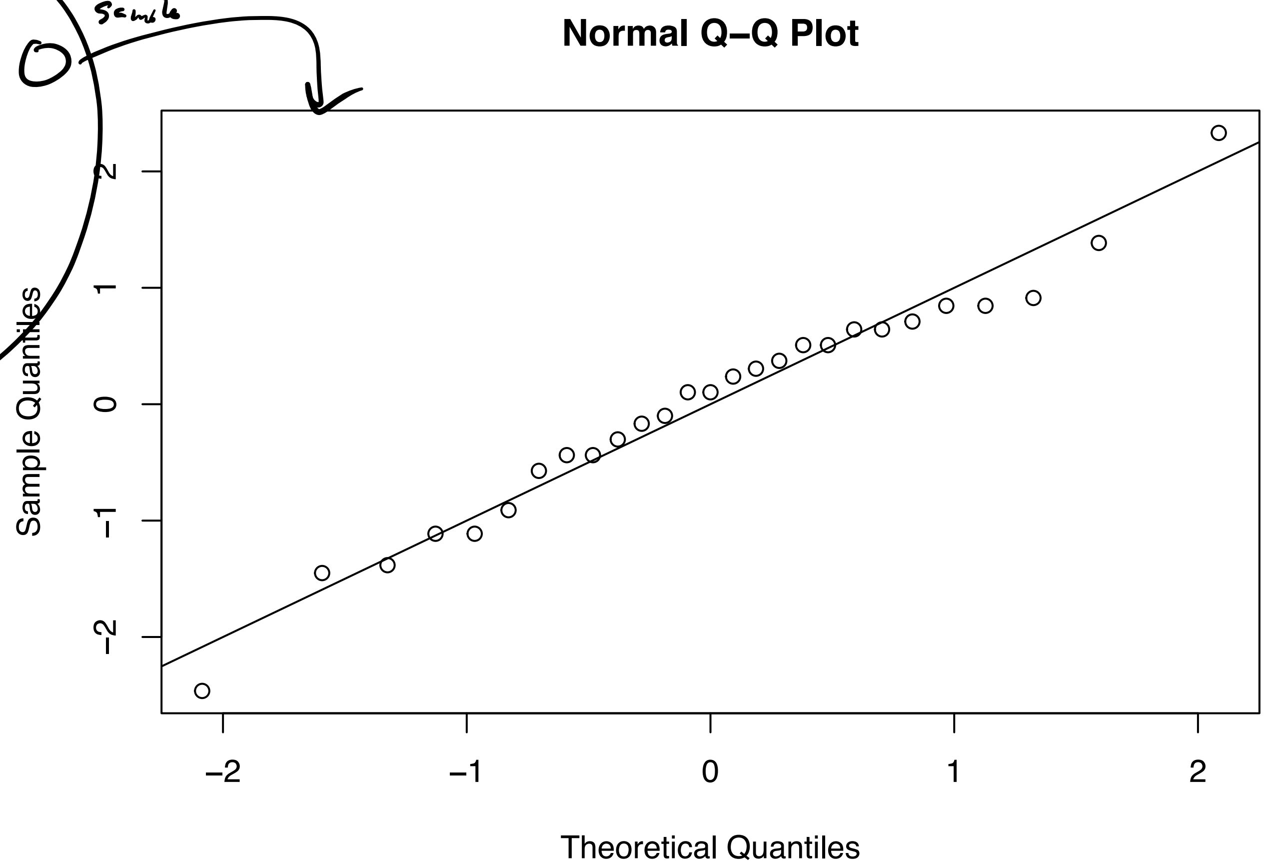
```r
gr <- c(1.66, 1.61, 1.62, 1.69, 1.58, 1.43, 1.66,
        1.69, 1.58, 1.20, 1.52, 1.60, 1.55, 1.67,
        1.77, 1.50, 1.64, 1.54, 1.40, 1.36, 1.50,
        1.40, 1.35, 1.48, 1.64, 1.91, 1.70)
```

What is the true mean of B/A? Could it be the golden ratio?!?

Check if B/A measurements come from a Normal distribution.

```r
qqnorm(scale(gr))
abline(0,1)
```





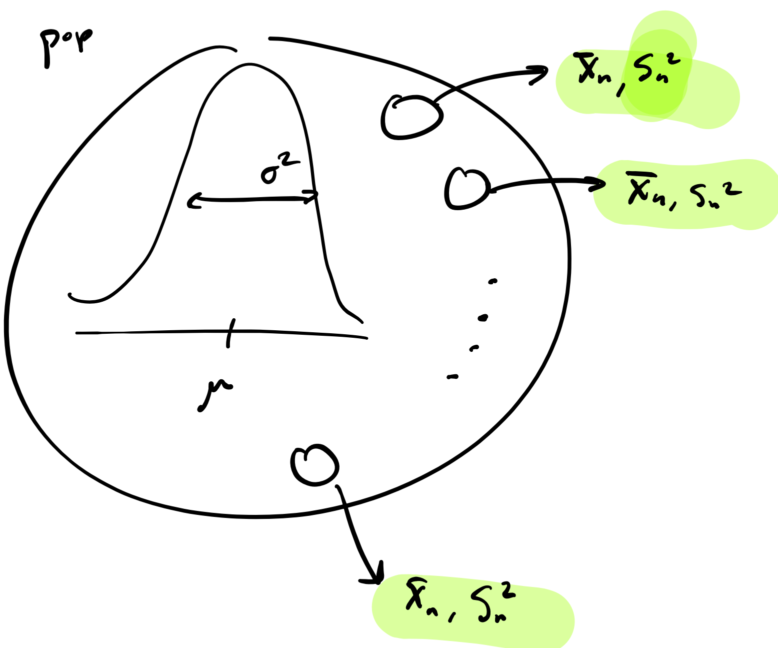
Estimation

Based on $X_1, \dots, X_n$, define the sample statistics

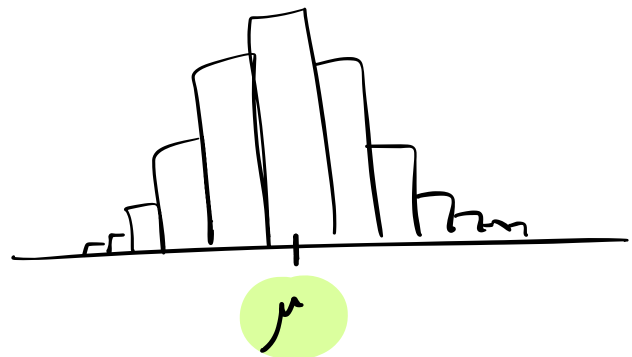
- $\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$
- $S_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X}_n)^2$

Then $\bar{X}_n$ and S_n^2 are unbiased estimators of μ and σ^2, respectively.

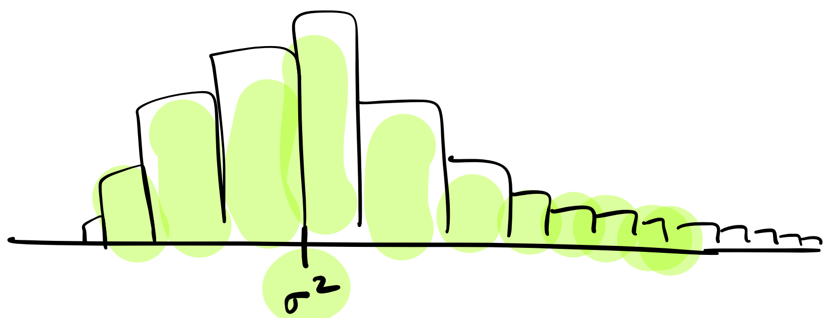
S_n = sample standard deviation



Histogram of $\bar{X}_n$



Histogram of S_n^2



"Expected Value": Average of a random variable.

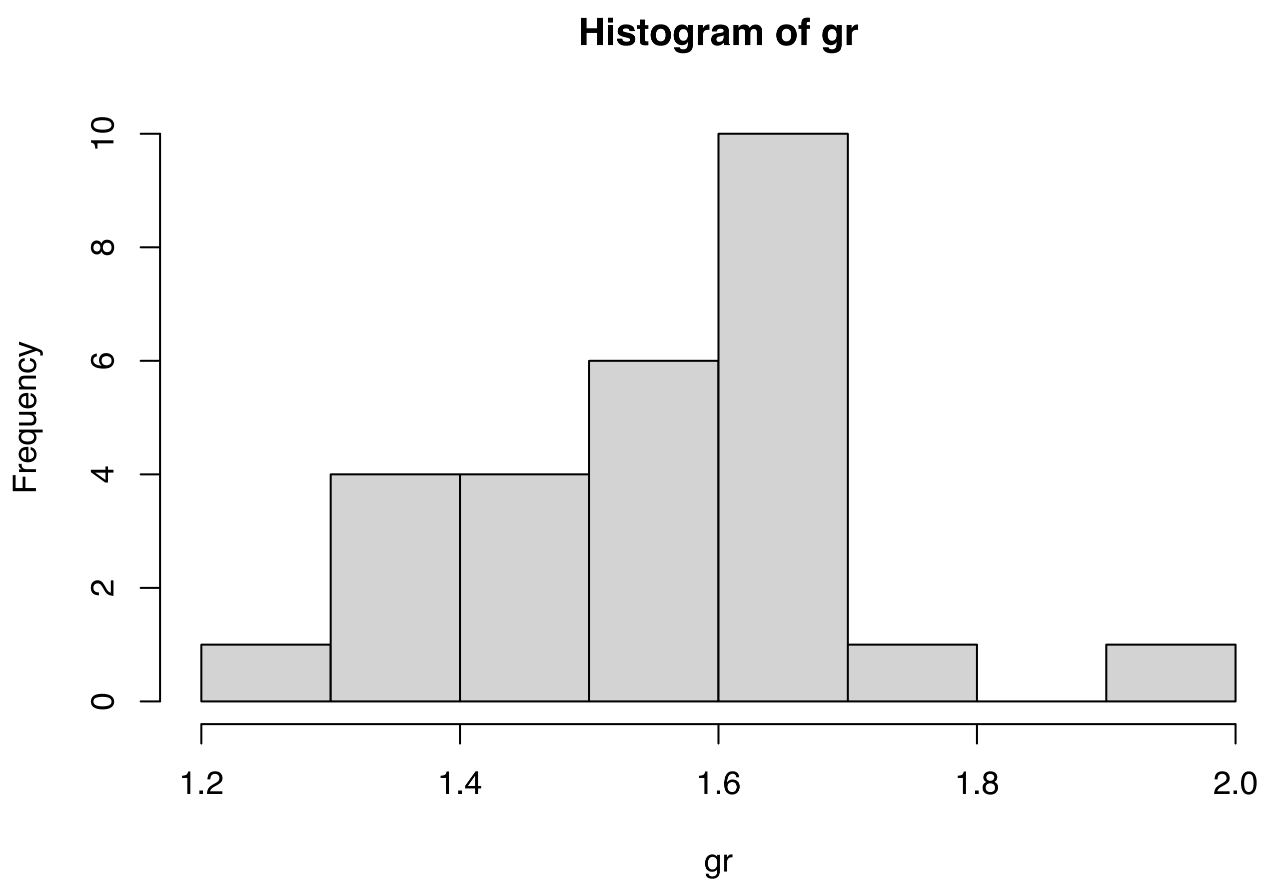
Unbiasedness

$$\boxed{\begin{aligned} \mathbb{E}\,\bar{X}_n &= \mu \\ \mathbb{E}\,S_n^2 &= \sigma^2 \end{aligned}}$$

Golden ratio example (cont):

We have $\bar{X}_n = \texttt{mean(gr)} = 1.565$ and $S_n^2 = \texttt{var(gr)} = 0.0219$.

```
hist(gr)
```



Important sampling distribution results

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}}$$

$$T = \frac{\bar{X}_n - \mu}{S_n/\sqrt{n}}$$

data values we will observe

independent

population

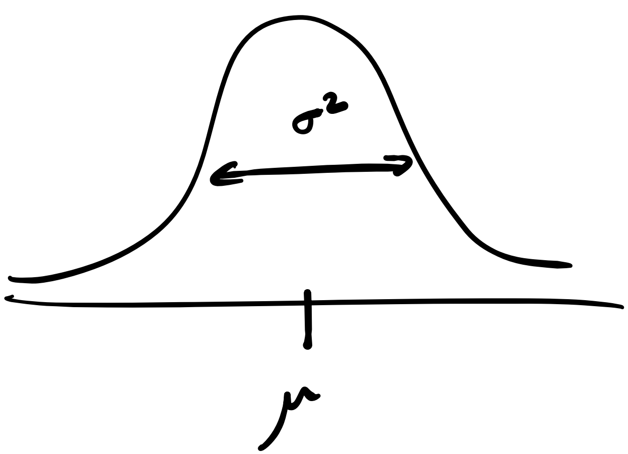
mean

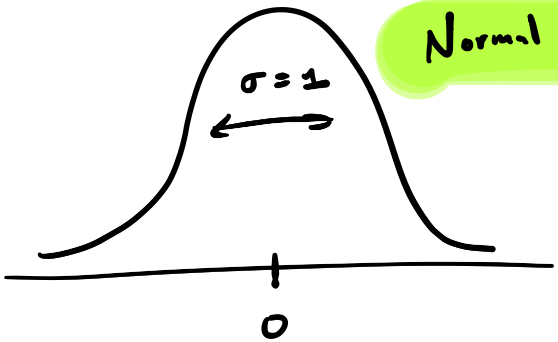
variance

Provided $X_1, \ldots, X_n \overset{\text{ind}}{\sim} \text{Normal}(\mu, \sigma^2)$, we have

distributed as / "follows"

- $\dfrac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \sim \text{Normal}(0, 1)$
- $\dfrac{(n-1)S_n^2}{\sigma^2} \sim \chi^2_{n-1}$ ← Chi-squared dist. with n-1 degrees of freedom n-1
- $\dfrac{\bar{X}_n - \mu}{S_n/\sqrt{n}} \sim t_{n-1}$ ← t-dist with d.f. n-1.



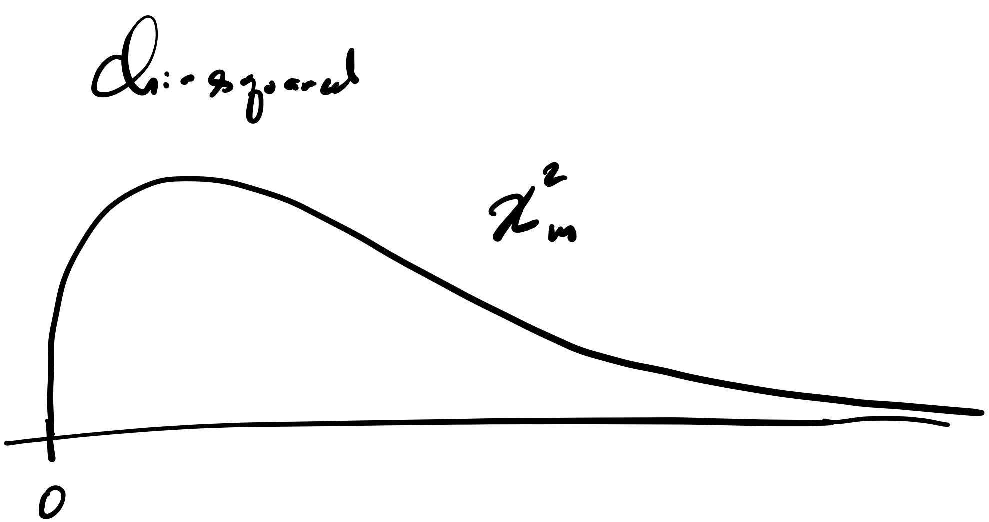


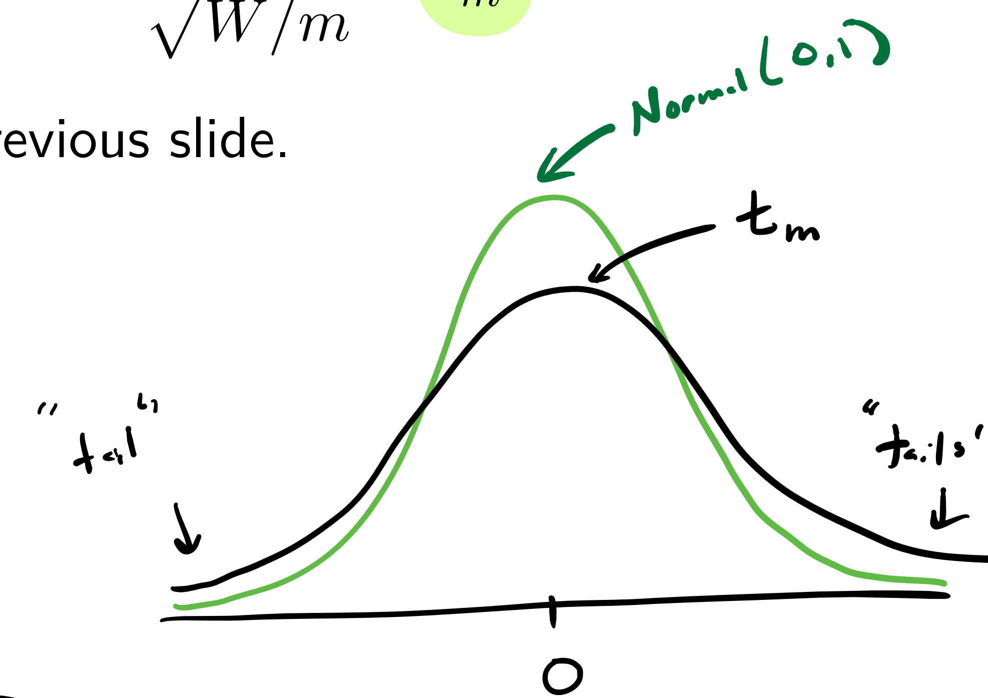
Normal(0,1) "Standard Normal"

Discuss: Anatomy of chi-square and t random variables

- $Z_1, \dots, Z_m \overset{\text{ind}}{\sim} \text{Normal}(0,1) \implies Z_1^2 + \cdots + Z_m^2 \sim \chi^2_m$.
- $Z \sim \text{Normal(0,1)} \perp\!\!\!\perp W \sim \chi^2_m \implies \dfrac{Z}{\sqrt{W/m}} \sim t_m$.

Relate these to the results on the previous slide.

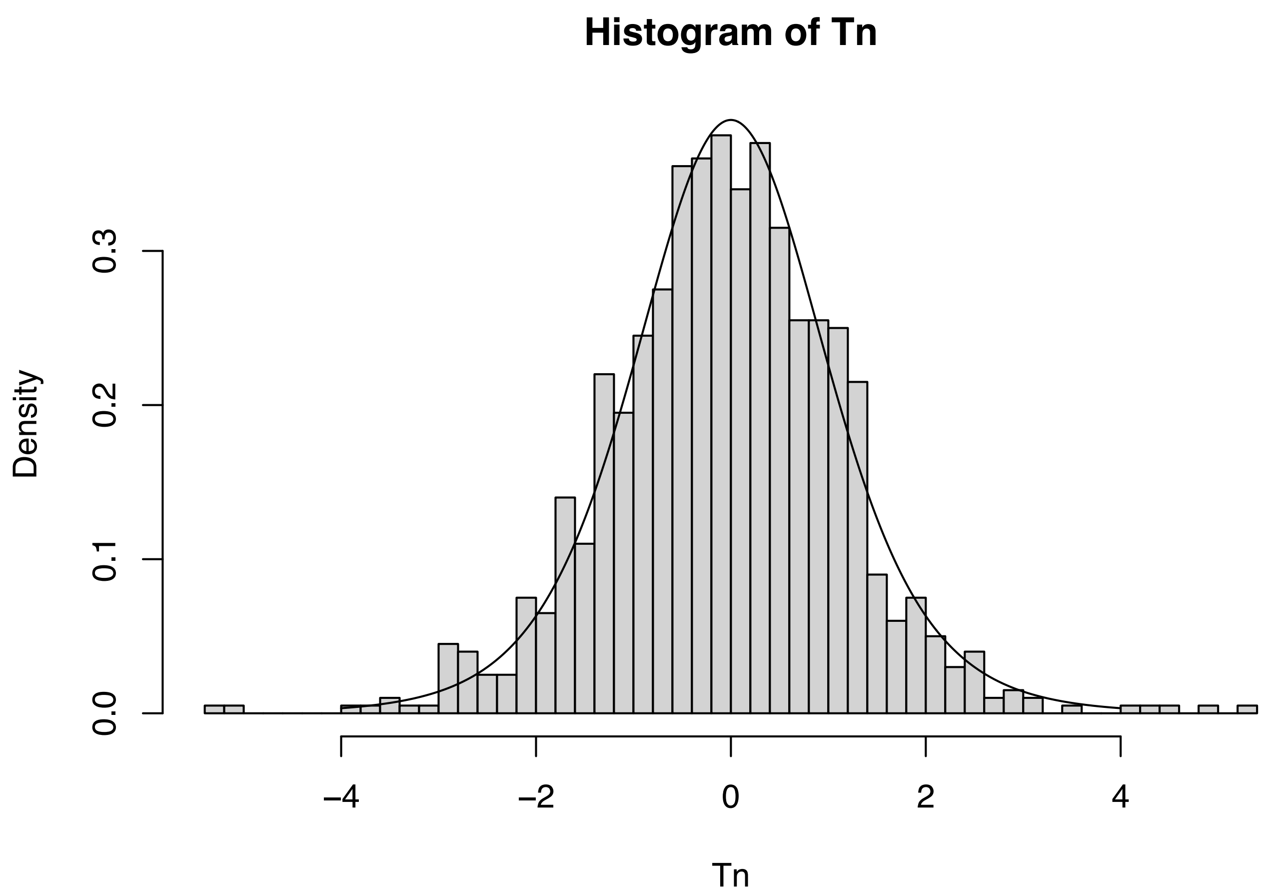




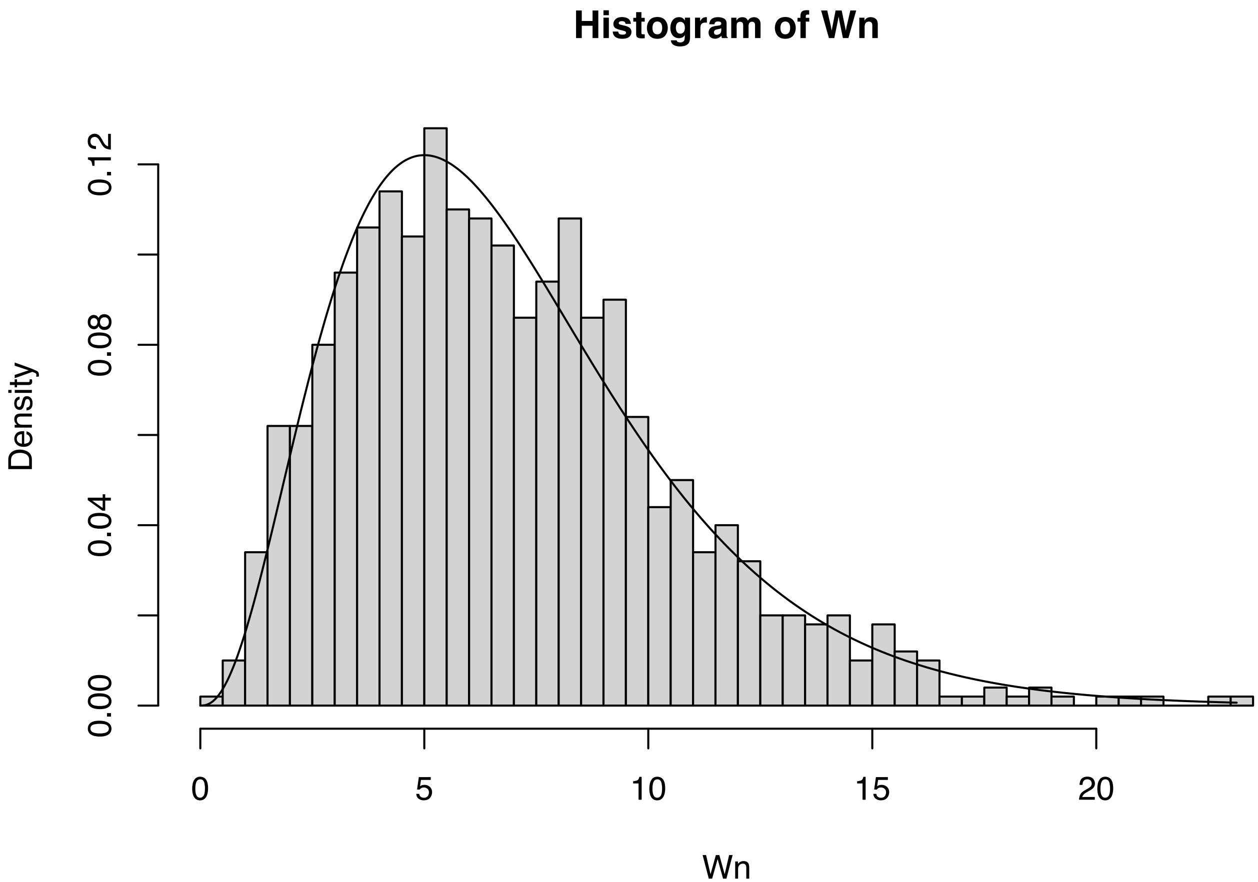
Simulation illustrating sampling distribution results:

```
sims <- 1000
mu <- 1
sigma <- 1/2
n <- 8
Tn <- numeric(sims)
Wn <- numeric(sims)
for(s in 1:sims){

  X <- rnorm(n,mu,sigma)
  sn <- sd(X)
  xbar <- mean(X)
  Tn[s] <- sqrt(n)*(xbar - mu) / sn
  Wn[s] <- (n-1)*sn^2 / sigma^2

}
```

```r
hist(Tn,freq = FALSE,breaks = 50)
x <- seq(-4,4,length = 500)
lines(dt(x,n-1)~x)
```



```r
hist(Wn,freq = FALSE,breaks = 50)
x <- seq(0,max(Wn),length = 500)
lines(dchisq(x,n-1)~x)
```



Confidence intervals for the mean and variance

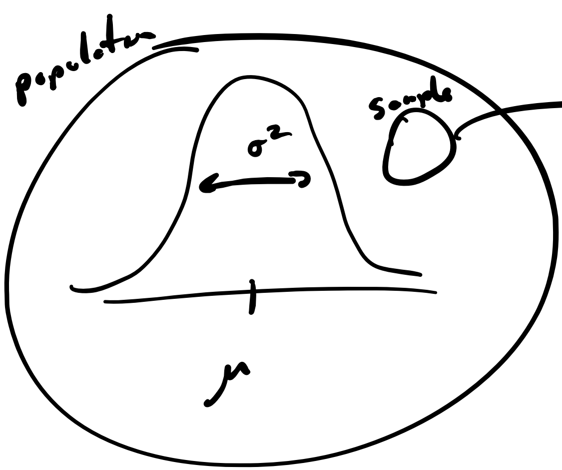
Try to estimate μ or σ^2 based on $\bar{X}_n$ and S_n^2.

The sampling distribution results give $(1-\alpha)100\%$ CIs as

1 - $\bar{X}_n \pm t_{n-1,\alpha/2} \dfrac{S_n}{\sqrt{n}}$ for μ.
- $\left(\dfrac{(n-1)S_n^2}{\chi^2_{n-1,\alpha/2}}, \dfrac{(n-1)S_n^2}{\chi^2_{n-1,1-\alpha/2}} \right)$ for σ^2.

Exercise: Derive the above.

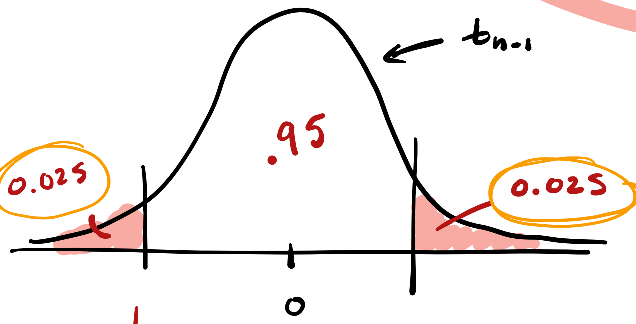
1



$\bar{X}_n, S_n^2$

$$\frac{\bar{X}_n - \mu}{S_n/\sqrt{n}} \sim t_{n-1}$$

$\alpha = 0.05$



$-t_{n-1,\,0.025}$ $\qquad$ $\underbrace{t_{n-1,\,0.025}} = qt(\qquad)$

$$P\left(-t_{n-1,\,0.025} < \frac{\bar{X}_n - \mu}{S_n/\sqrt{n}} < t_{n-1,\,0.025}\right) = 0.95$$

Get a 95% C.I. for μ:

$$P\left(-S_n/\sqrt{n}\; t_{n-1,\,0.025} < \bar{X}_n - \mu < \frac{S_n}{\sqrt{n}}\, t_{n-1,\,0.025}\right) = 0.95$$

$\Leftrightarrow$

$$P\left(-\bar{X}_n - \frac{S_n}{\sqrt{n}}\, t_{n-1,\,0.025} < -\mu < -\bar{X}_n + \frac{S_n}{\sqrt{n}}\, t_{n-1,\,0.025}\right) = 0.95$$

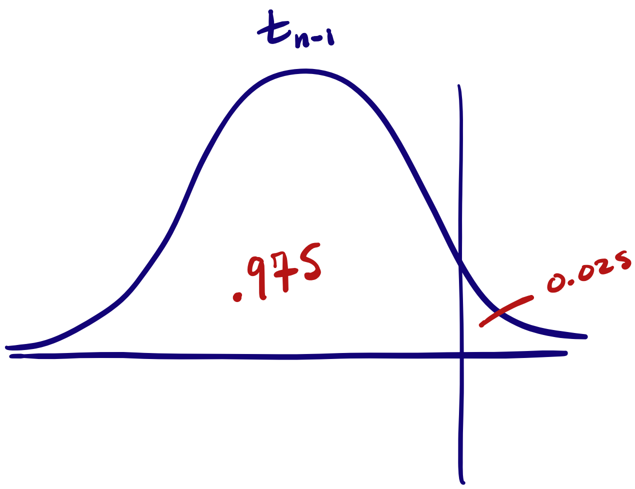
$\Leftrightarrow$

$$P\left(\underbrace{\bar{X}_n + \frac{S_n}{\sqrt{n}}\, t_{n-1,\,0.025}}_{\text{upper}} > \mu > \underbrace{\bar{X}_n - \frac{S_n}{\sqrt{n}}\, t_{n-1,\,0.025}}_{\text{lower}}\right) = 0.95$$

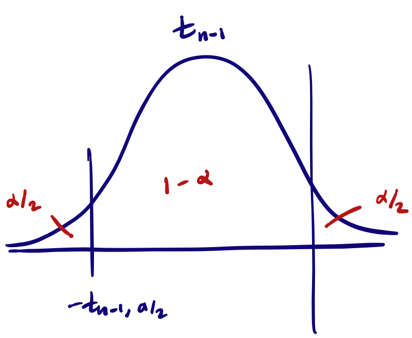
So 95% C.I. is $\left[\bar{X}_n - \frac{S_n}{\sqrt{n}}\, t_{n-1,\,0.025},\ \bar{X}_n + \frac{S_n}{\sqrt{n}}\, t_{n-1,\,0.025}\right]$

or

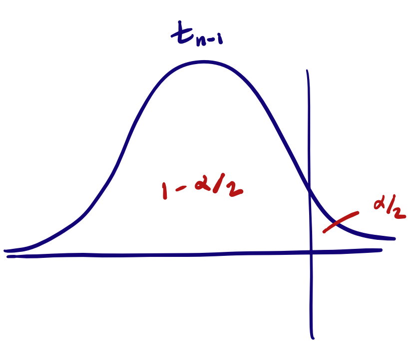
$$\bar{X}_n \pm \frac{S_n}{\sqrt{n}} t_{n-1,\,0.025}.$$



$$t_{n-1,0.025} = qt(.975,\ n-1)$$

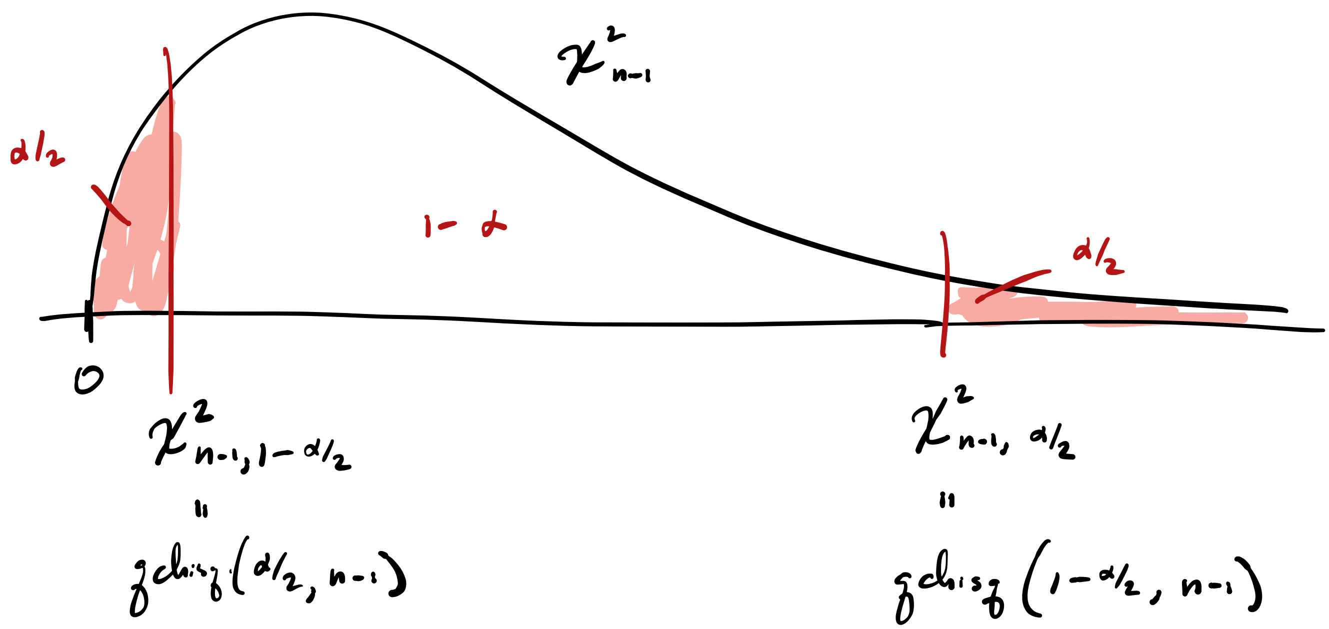


$$t_{n-1,\alpha/2} = qt(1-\alpha/2,\ n-1)$$



$$t_{n-1,\alpha/2} = qt(1-\alpha/2,\ n-1)$$

$$\left(\frac{(n-1)S_n^2}{\chi^2_{n-1,\alpha/2}}, \frac{(n-1)S_n^2}{\chi^2_{n-1,1-\alpha/2}}\right) \text{ for } \sigma^2.$$



Golden ratio example (cont):

Build 95% CIs for population mean and variance of B/A values:

```
alpha <- 0.05
n <- length(gr)

lomu <- mean(gr) - qt(1-alpha/2,n-1) * sd(gr)/sqrt(n)
upmu <- mean(gr) + qt(1-alpha/2,n-1) * sd(gr)/sqrt(n)

losgs <- (n-1) * var(gr) / qchisq(1-alpha/2,n-1)
upsgs <- (n-1) * var(gr) / qchisq(alpha/2,n-1)
```

The 95% CI for μ is (1.506,1.623). For σ^2 it is (0.014,0.041).

"null value"

$H_0: \mu \geq \mu_0$

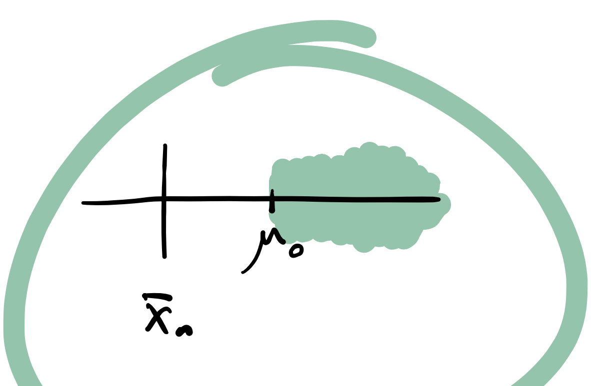
$H_1: \mu < \mu_0$

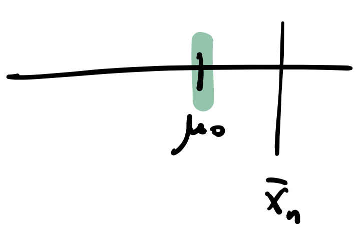
$H_0: \mu = \mu_0$

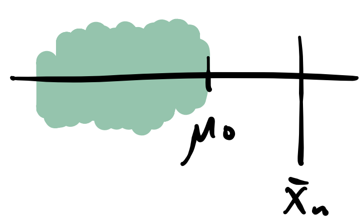
$H_1: \mu \neq \mu_0$

$H_0: \mu \leq \mu_0$

$H_1: \mu > \mu_0$







Testing hypotheses about the mean

two sided.

Consider testing hypotheses about μ of the form

1-sided

1-sided

$$H_0\colon \mu \geq \mu_0 \quad H_1\colon \mu < \mu_0 \qquad \text{or} \qquad H_0\colon \mu = \mu_0 \quad H_1\colon \mu \neq \mu_0 \qquad \text{or} \qquad H_0\colon \mu \leq \mu_0 \quad H_1\colon \mu > \mu_0.$$

Reject or fail to reject H_0 based on the value of the test statistic

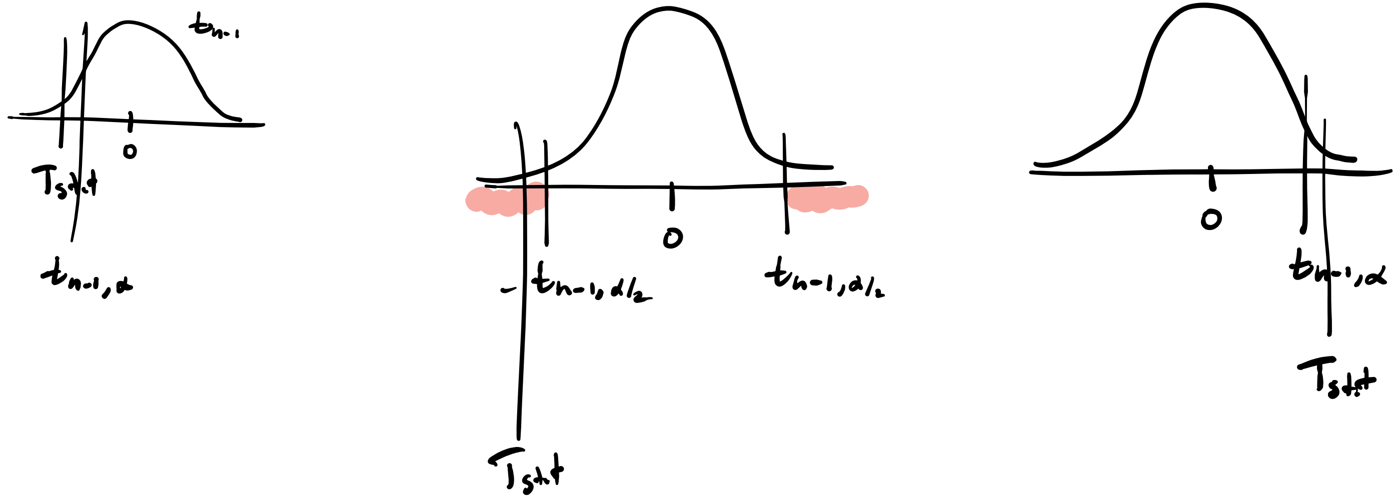
$$T_{\text{stat}} = \frac{\bar{X}_n - \mu_0}{S_n/\sqrt{n}}.$$

Rejection rules for the above at significance level α are

$$T_{\text{stat}} < -t_{n-1,\alpha} \qquad \text{or} \qquad |T_{\text{stat}}| > t_{n-1,\alpha/2} \qquad \text{or} \qquad T_{\text{stat}} > t_{n-1,\alpha}.$$

The corresponding p-values are, with $T \sim t_{n-1}$, the probabilities

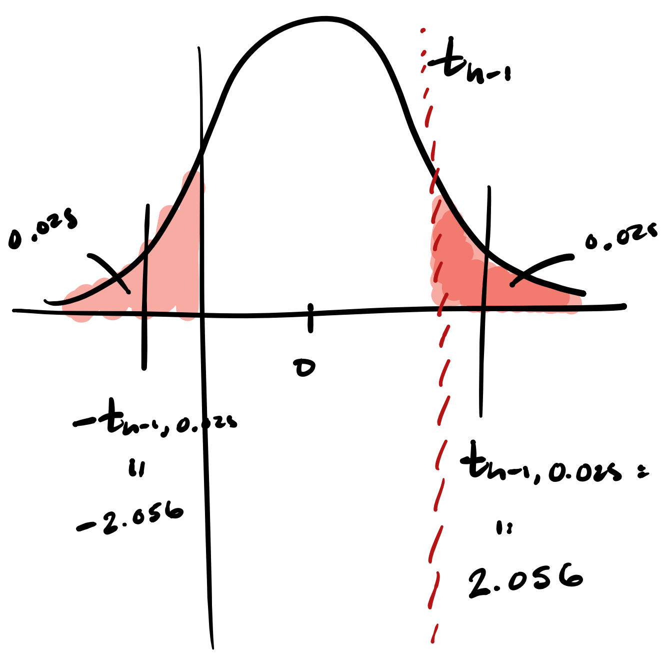
$$P(T < T_{\text{stat}}) \qquad \text{or} \qquad 2 \times P(T > |T_{\text{stat}}|) \qquad \text{or} \qquad P(T > T_{\text{stat}}).$$



Test $\quad H_0: \mu = 1.618 \quad (\mu_0)$ $\quad$ at $\quad$ 5% significance level.

$\quad\quad H_1: \mu \neq 1.618$

$$T_{stat} = \frac{\bar{X}_n - \mu_0}{S_n/\sqrt{n}} = \frac{\bar{X}_n - 1.618}{S_n/\sqrt{n}} = -1.866$$



$t_{n-1,0.025} = qt(.975, n-1)$
$= 2.056$

$T_{stat} \quad -1.866$

p-value $= 2 * 0.037$
$= .074$

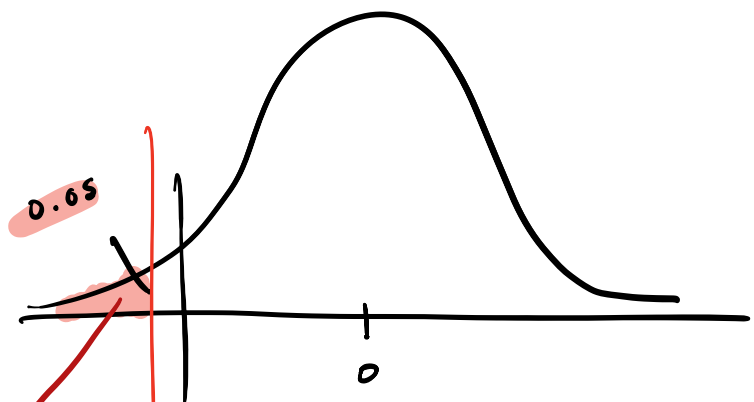
We fail to reject $H_0: \mu = 1.618$.

$H_0: \mu \geq 1.618$

$H_1: \mu < 1.618$

$\bar{x}_n = 1.56$

$$T_{stat} = \frac{\bar{x}_n - 1.618}{s/\sqrt{n}} = -1.866$$



p-value = 0.037

$$-t_{n-1,\,0.05} = -qt(.95, n-1) = -1.71$$

T_{stat}

$pt(T_{stat}, n-1) = 0.037$

<u>p-value</u>: smallest α at which you reject H_0.

Golden ratio example (cont):

Test H_0: $\mu = 1.618$ vs H_1: $\mu \neq 1.618$ at $\alpha = 0.05$ based on data.

```
alpha <- 0.05
Tstat <- (mean(gr) - 1.618) / (sd(gr) / sqrt(n))
abs(Tstat) > qt(1-alpha/2,n-1)
```

```
[1] FALSE
```

Fail to reject H_0 since T_{stat} = -1.866 is smaller in absolute value than $t_{n-1,\alpha/2}$ = 2.056.

```
pval <- 2*(1 - pt(abs(Tstat),n-1))
```

Equivalently, the p-value, which is 0.073, is greater than $\alpha = 0.05$.

The `t.test()` function in R

The function `t.test()` tests H_0: $\mu = 0$ vs H_1: $\mu \neq 0$ by default.

```
t.test(gr)
```

```

	One Sample t-test

data:  gr
t = 54.902, df = 26, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 1.506228 1.623401
sample estimates:
mean of x
 1.564815
```

The `t.test()` function in R

Now test H_0: $\mu = 1.618$ versus H_1: $\mu \neq 1.618$, ask for 99% CI.

```r
t.test(gr, mu = 1.618, conf.level = 0.99)
```

```
	One Sample t-test

data:  gr
t = -1.866, df = 26, p-value = 0.07336
alternative hypothesis: true mean is not equal to 1.618
99 percent confidence interval:
 1.485616 1.644013
sample estimates:
mean of x 
 1.564815 
```

The `t.test()` function in R

Now test H_0: $\mu \leq 1.618$ versus H_1: $\mu > 1.618$.

```
t.test(gr, mu = 1.618, alternative = "greater")
```

```

    One Sample t-test

data:  gr
t = -1.866, df = 26, p-value = 0.9633
alternative hypothesis: true mean is greater than 1.618
95 percent confidence interval:
 1.516202      Inf
sample estimates:
mean of x 
 1.564815 
```

Testing hypotheses about the variance

optional

Consider testing hypotheses about σ^2 of the form

$$\begin{array}{l} H_0\colon \sigma^2 \geq \sigma_0^2 \\ H_1\colon \sigma^2 < \sigma_0^2 \end{array} \quad \text{or} \quad \begin{array}{l} H_0\colon \sigma^2 \leq \sigma_0^2 \\ H_1\colon \sigma^2 > \sigma_0^2 \end{array}$$

Reject or fail to reject H_0 based on the value of the test statistic

$$W_{\text{stat}} = \frac{(n-1)S_n^2}{\sigma_0^2}.$$

Rejection rules for the above at significance level α are

$$W_{\text{stat}} < \chi^2_{n-1,1-\alpha} \quad \text{or} \quad W_{\text{stat}} > \chi^2_{n-1,\alpha}$$

The corresponding p-values are, with $W \sim \chi^2_{n-1}$, the probabilities

$$P(W < W_{\text{stat}}) \quad \text{or} \quad P(W > W_{\text{stat}}).$$

Golden ratio example (cont):

optional

Test H_0: $\sigma^2 \geq 0.03$ vs H_1: $\sigma^2 < 0.03$ at $\alpha = 0.05$ based on data.

```
alpha <- 0.05
Wstat <- (n-1)*var(gr) / 0.03
Wstat < qchisq(alpha,n-1)
```

```
[1] FALSE
```

FTR H_0 since $W_{\text{stat}} = 19.009$ is not less than $\chi^2_{n-1,1-\alpha} = 15.379$.

```
pval <- pchisq(Wstat,n-1)
```

Equivalently, the p-value, which is 0.164, is greater than $\alpha = 0.05$.

Sample size calculations

C.I. for μ: $\bar{X}_n \pm t_{n-1,\alpha/2} \frac{S_n}{\sqrt{n}}$

Margin of Error

$2\, t_{n-1,\alpha/2} \frac{S_n}{\sqrt{n}}$

$n = ?$

$\bar{X}_n - t_{n-1,\alpha/2} \frac{S_n}{\sqrt{n}}$ $\quad \bar{X}_n \quad$ $\bar{X}_n + t_{n-1,\alpha/2} \frac{S_n}{\sqrt{n}}$

We can choose a sample size based on the desired:

a. Width of a confidence interval.
b. Power of a test to reject H_0 when it is false.

guess the value of this from previous.

Scientist wants this margin of error.

a. Find smallest n such that $t_{n-1,\alpha/2} \frac{S_n}{\sqrt{n}} < M$

replace with $z_{\alpha/2}$

Let $\hat{\sigma}$ is a guess of σ. Then find smallest n such that

$$z_{\alpha/2} \frac{\hat{\sigma}}{\sqrt{n}} < M^*$$

$$z_{\alpha/2} \frac{\hat{\sigma}}{M^*} < \sqrt{n}$$

$$\left(\frac{z_{\alpha/2} \, \hat{\sigma}}{M^*} \right)^2 < n$$

Sample size required to achieve desired CI width

A CI for μ takes the form $\bar{X}_n \pm M$, where

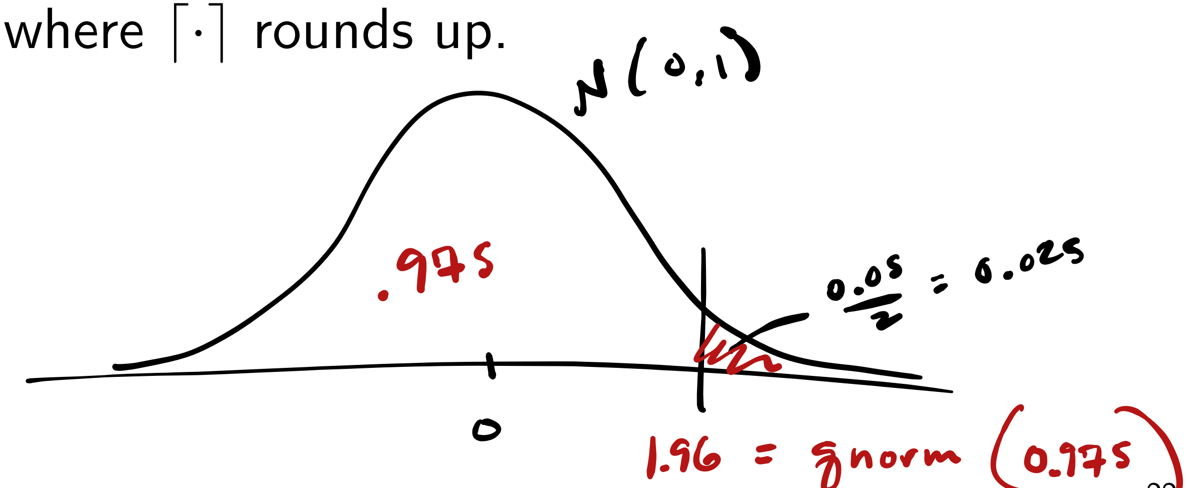
- $M = z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$ if σ is known $\quad \bar{X}_n \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
- $M = t_{n-1,\alpha/2} \dfrac{S_n}{\sqrt{n}}$ if σ is unknown $\quad \bar{X}_n \pm t_{n-1,\alpha/2} \frac{S_n}{\sqrt{n}}$

For ease, use the "σ-known" version.

If one wants $M \leq M^*$, find smallest n such that $z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}} \leq M^*$.

So take $n = \left\lceil \left(\dfrac{z_{\alpha/2}\sigma}{M^*} \right)^2 \right\rceil$, where $\lceil \cdot \rceil$ rounds up.

Must put in a guess for σ.



Golden ratio example (cont):

95% C.I. based on $n=27$

$[1.506, 1.623]$

Use $s_n = 0.148$ as our guess of σ.

$\alpha = 0.05$ $\quad M^* = 0.04$

$$n \geq \left\lceil \left(\frac{z_{\frac{0.05}{2}} \cdot 0.148}{.04} \right)^2 \right\rceil = 53$$

Find n required to make the 95% CI for μ no wider than 0.08.

```
alpha <- 0.05
M <- 0.08/2
sigma_guess <- sd(gr)
nr <- ceiling((qnorm(1-alpha/2) * sigma_guess / M)^2)
nr
```

```
[1] 53
```

Outcomes of Hypothesis Testing

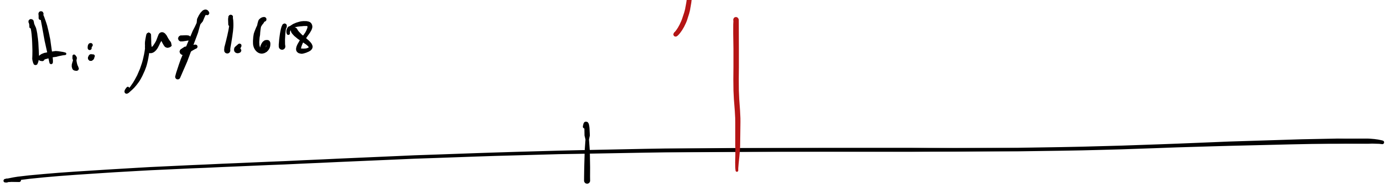
	H_0 True	H_0 False
Reject H_0	Type I error	Correct Decision
Fail to reject H_0	Correct Decision	Type II error

$$P(\text{Type I error}) \le \alpha$$

$H_0: \mu = 1.618$

$H_1: \mu \neq 1.618$

suppose $\mu = 1.7$



1.618 = Golden Ratio

Statistical Power: The probability of reject H_0 at some particular value of the parameter.

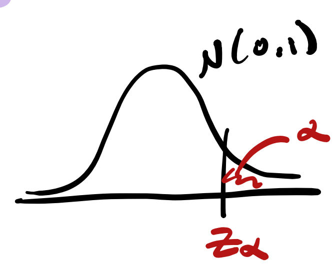
$H_0: \mu \le 1.618$

$H_1: \mu > 1.618$

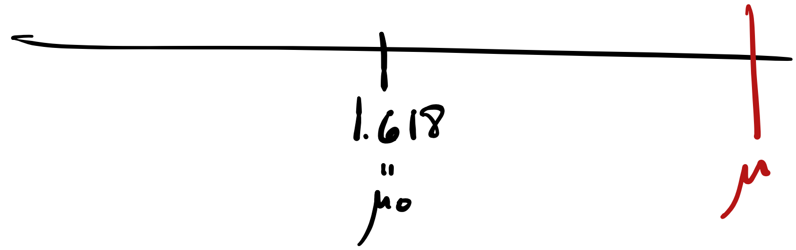
Assume σ is known.

$$Z_{test} = \frac{\bar{X}_n - \underbrace{1.618}_{\mu_0}}{\sigma/\sqrt{n}}$$

Reject H_0 if $Z_{test} > Z_\alpha$



Power function



$$\gamma(\mu) = P\Big(\underbrace{Z_{test} > z_\alpha}\ \underbrace{\text{if the true mean is } \mu} \Big)$$

"gamma"

$$= P_\mu\left(\frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}} > z_\alpha \right)$$

treat μ like the true mean

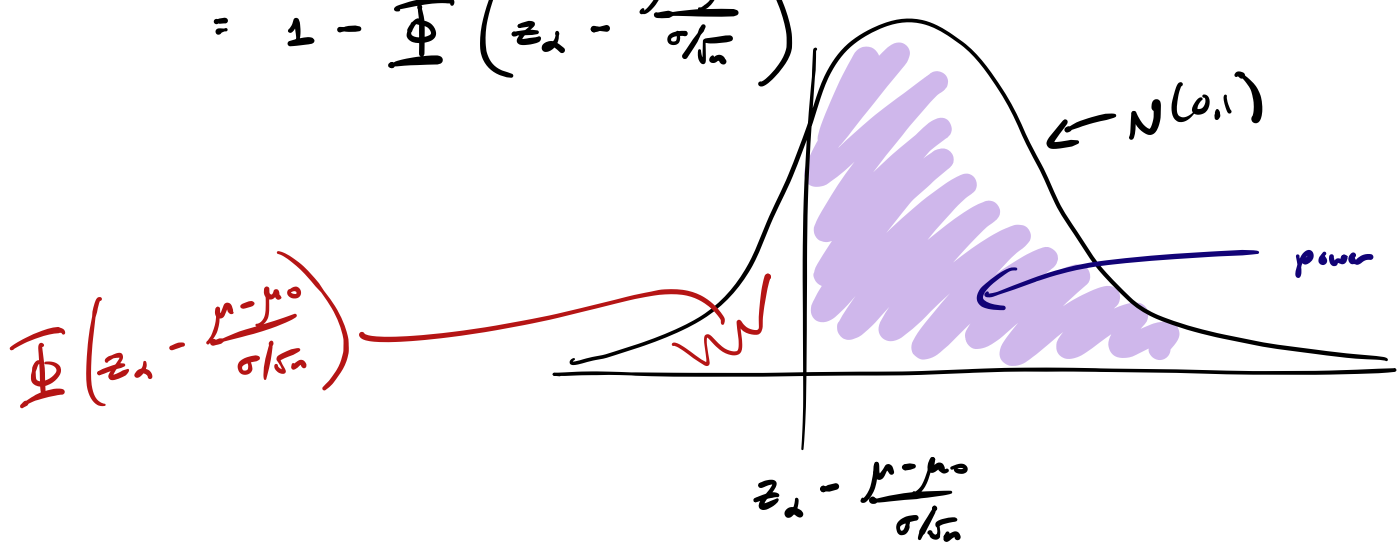
add / subtract true mean μ

$$= P_\mu\left(\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} + \frac{\mu - \mu_0}{\sigma/\sqrt{n}} > z_\alpha \right)$$

$\sim N(0,1)$

$$= P\left(Z > z_\alpha - \frac{\mu - \mu_0}{\sigma/\sqrt{n}} \right), \quad Z \sim N(0,1)$$

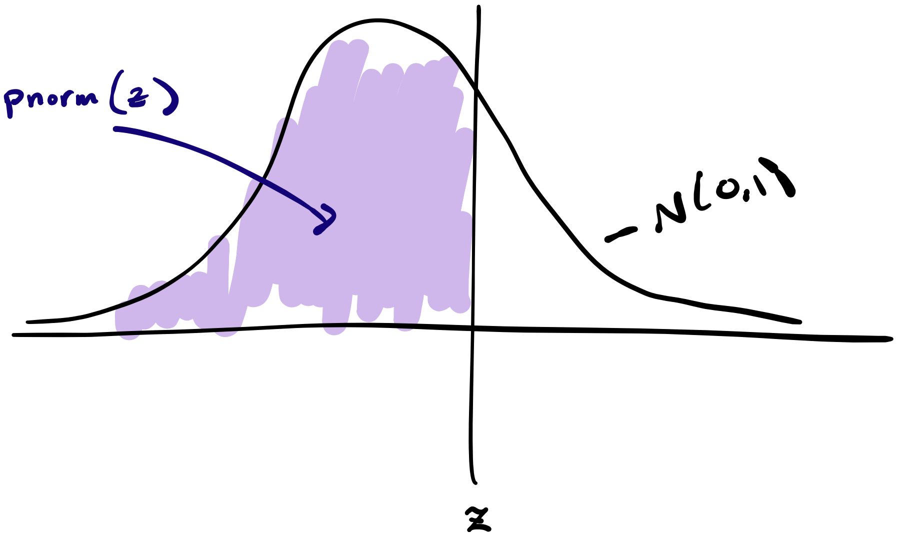
$$= 1 - \Phi\left(z_\alpha - \frac{\mu - \mu_0}{\sigma/\sqrt{n}} \right)$$

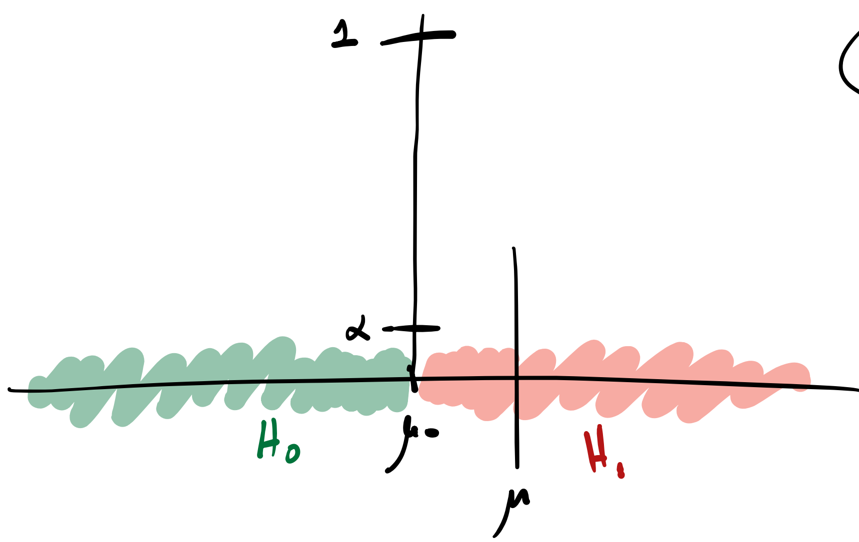


Standard Normal Cumulative Dist. Function

$\Phi(z)$ = pnorm(z)

"Phi"





$H_0: \mu \le \mu_0$

$H_1: \mu > \mu_0$

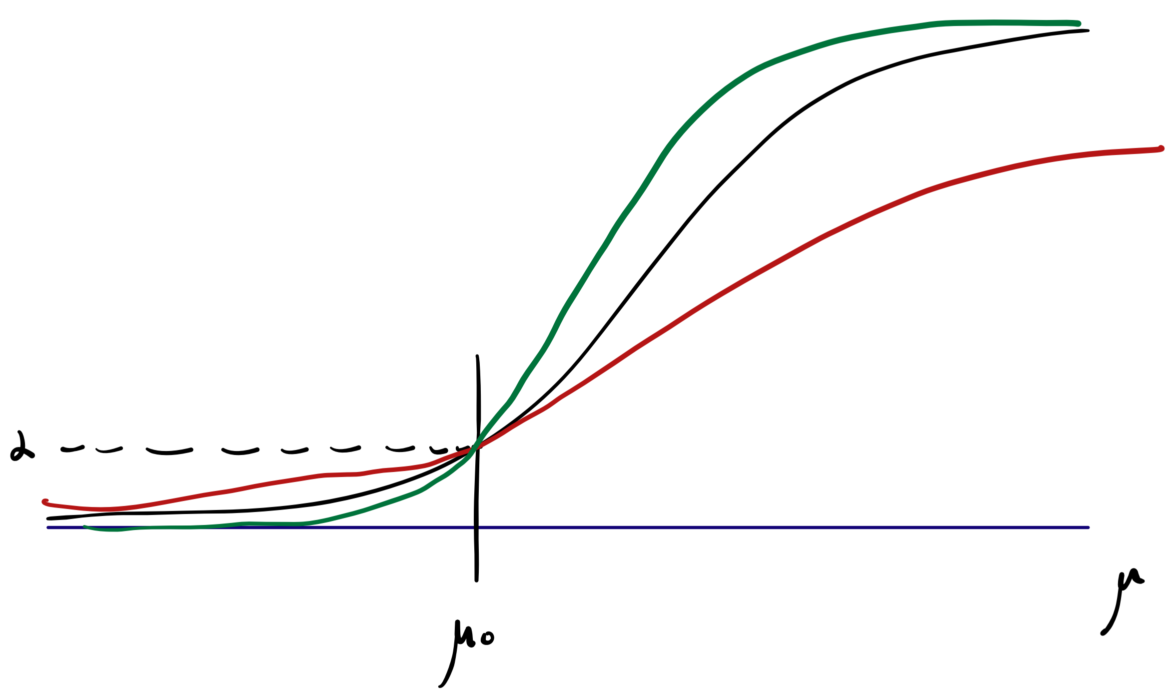
qnorm$(1-\alpha)$

$$\gamma(\mu) = 1 - \Phi\left(z_\alpha - \frac{\mu-\mu_0}{\sigma/\sqrt{n}}\right)$$

pnorm$(\qquad\qquad)$

$H_0: \mu \le \mu_0$

$\gamma(\mu)$ $H_1: \mu > \mu_0$



Sample size required to achieve desired power

The power of a test is the probability with which it rejects H_0.

For tests of H_0 concerning the mean μ we write the power as

$$\gamma(\mu) = P(\text{Reject } H_0 \text{ when true mean is } \mu) = P_\mu(\text{Reject } H_0).$$

So the power depends on the true value of μ, i.e. is a function of μ.

Exercise: Derive the power functions for the tests of

$$\begin{array}{l} H_0\colon \mu \geq \mu_0 \\ H_1\colon \mu < \mu_0 \end{array} \quad \text{and} \quad \begin{array}{l} H_0\colon \mu = \mu_0 \\ H_1\colon \mu \neq \mu_0 \end{array} \quad \text{and} \quad \begin{array}{l} H_0\colon \mu \leq \mu_0 \\ H_1\colon \mu > \mu_0 \end{array}$$

with the rejection rules

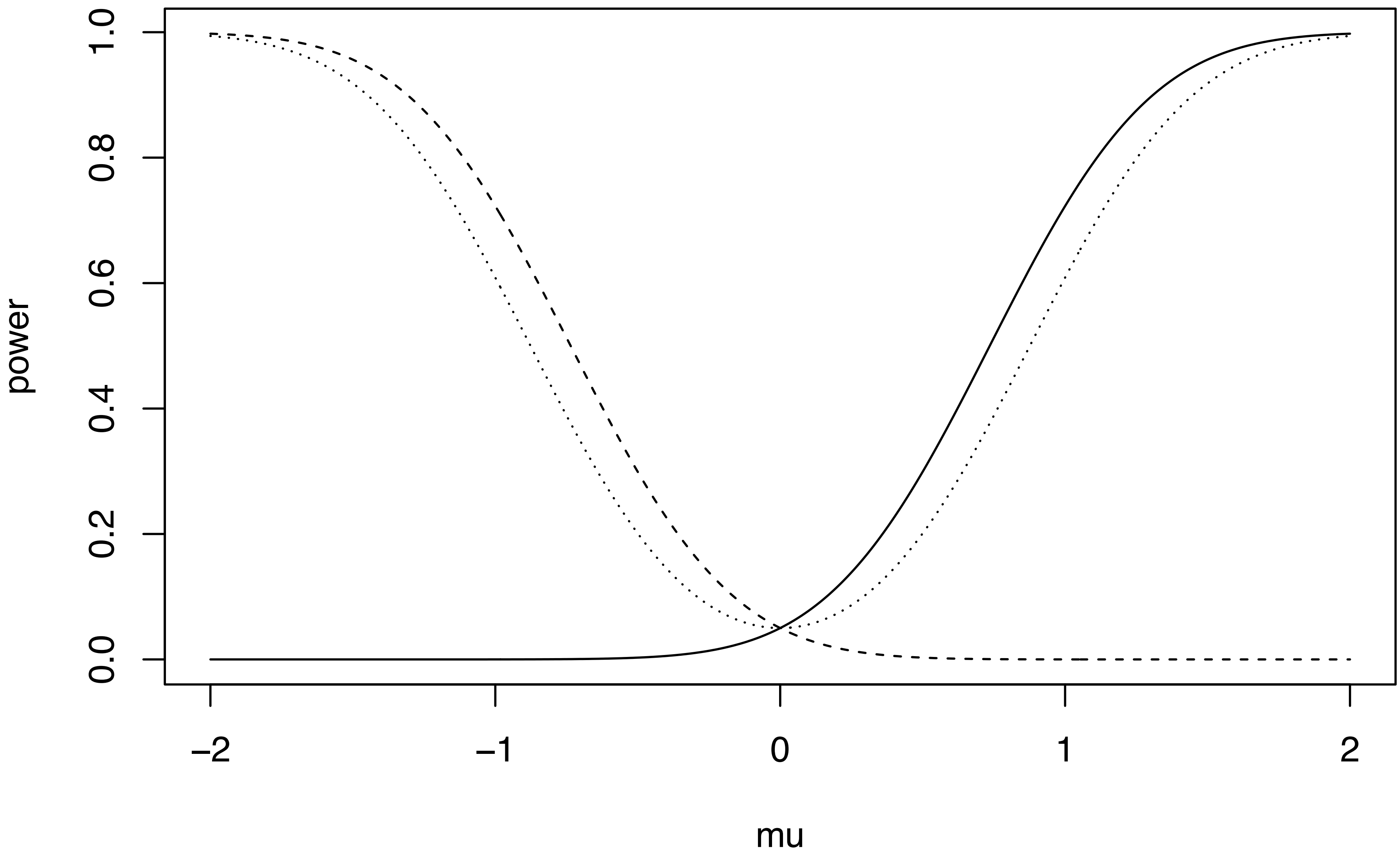
$$Z_{\text{stat}} < -z_\alpha \quad \text{and} \quad |Z_{\text{stat}}| > z_{\alpha/2} \quad \text{and} \quad Z_{\text{stat}} > z_\alpha,$$

respectively, where $Z_{\text{stat}} = \dfrac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}}$ (σ-known case).

Plot of power curves for right-, left-, and two-sided tests

```
alpha <- 0.05
sigma <- 1
n <- 5
mu0 <- 0
mu <- seq(-2,2,length=500)
za <- qnorm(1-alpha)
za2 <- qnorm(1-alpha/2)
d <- sqrt(n) * (mu - mu0) / sigma
rp <- 1 - pnorm(za - d)
lp <- pnorm(-za - d)
rp2 <- 1 - pnorm(za2 - d)
lp2 <- pnorm(-za2 - d)
tsp <- lp2 + rp2
```

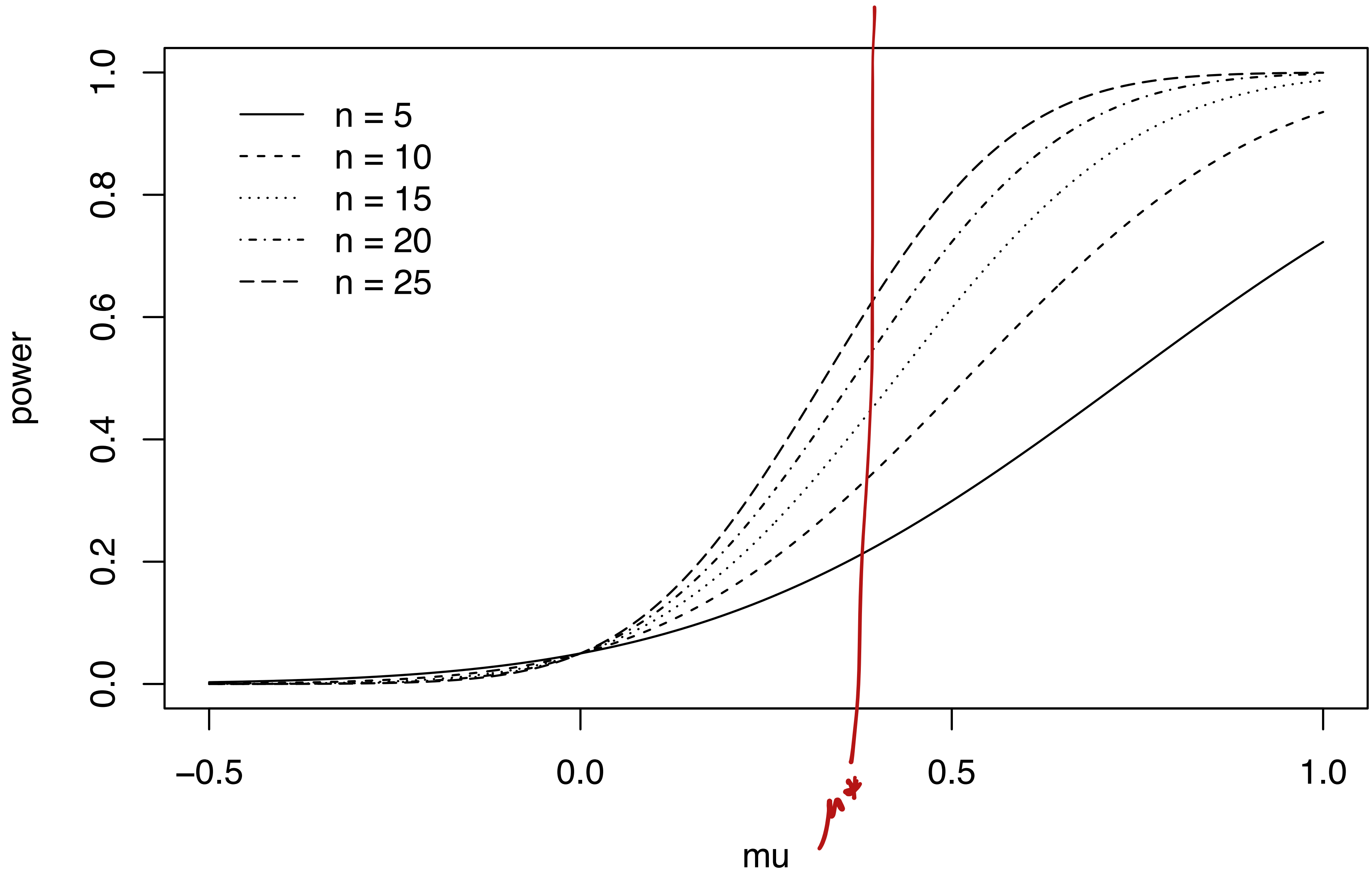
```r
plot(rp ~ mu, type = "l", ylab = "power", xlab = "mu")
lines(lp ~ mu, lty =2)
lines(tsp ~ mu, lty = 3)
```



Power curve for right-sided test at various sample sizes

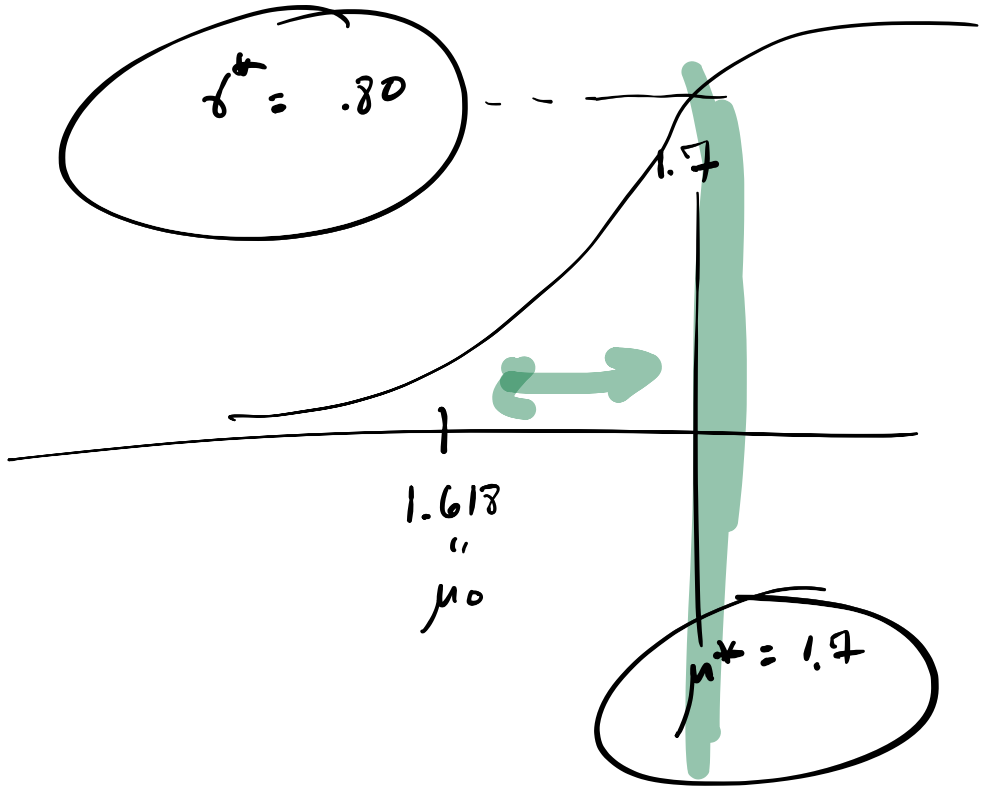
```
alpha <- 0.05
sigma <- 1
nn <- c(5,10,15,20,25)
mu0 <- 0
mu <- seq(-1/2,1,length=500)
za <- qnorm(1-alpha)
rp <- matrix(NA,500,length(nn))
for(j in 1:length(nn)){
  d <- sqrt(nn[j]) * (mu - mu0) / sigma
  rp[,j] <- 1 - pnorm(za - d)
}
```

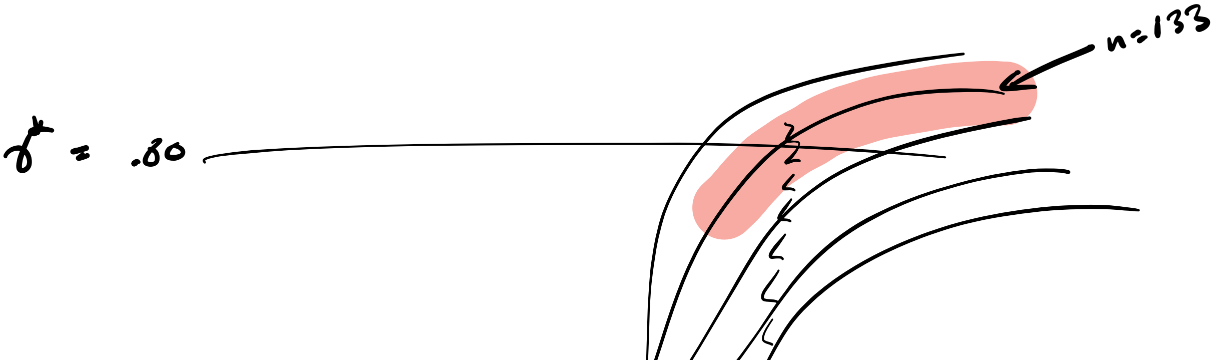
```r
plot(NA,xlim = range(mu), ylim = c(0,1), ylab = "power", xlab = "mu")
for(j in 1:length(nn)) lines(rp[,j] ~ mu, lty = j)
legend(x = min(mu), y = 1,legend = paste("n =",nn),lty = 1:length(nn),bty = "n")
```



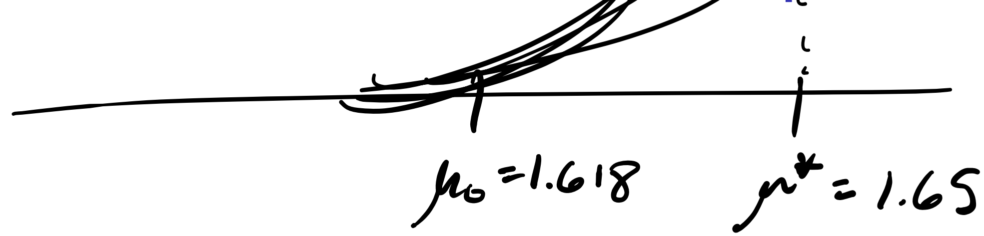
$H_0: \mu = 1.618$

$H_1: \mu > 1.618$





Sample size based on desired power



$\gamma^* = 0.80$

To find the smallest sample size guaranteeing a desired power:

1. Fix an alternative value μ^* and a desired power γ^*.
2. Set up the equation $\gamma(\mu^*) = \gamma^*$ and solve for n (then round up).

For our tests concerning μ when σ is known, we obtain:

- In the one-sided case $\boxed{n = \left\lceil \sigma^2 \left(\frac{z_\alpha + z_{\beta^*}}{\mu^* - \mu_0} \right)^2 \right\rceil}$.

$\beta^* = 1 - \gamma^*$

- In the two-sided case $n = \left\lceil \sigma^2 \left(\frac{z_{\alpha/2} + z_{\beta^*}}{\mu^* - \mu_0} \right)^2 \right\rceil$.

$z_\alpha =$ qnorm $(1 - \alpha)$

$z_{\beta^*} =$ qnorm $(1 - \beta^*) =$ qnorm (γ^*)

Exercise: Derive the sample size formula for the test of $H_0\colon \mu \leq \mu_0$ vs $H_1\colon \mu > \mu_0$ when σ is known.

$$\gamma(\mu) = 1 - \Phi\left(z_\alpha - \frac{\mu - \mu_0}{\sigma/\sqrt{n}}\right)$$

Write

$$\gamma^* = 1 - \Phi\left(z_\alpha - \frac{\mu^* - \mu_0}{\sigma/\sqrt{n}}\right)$$

solve for n. Gives

Golden ratio example (cont):

$\mu^* = 1.65$, $\gamma^* = 0.80$

$H_0: \mu \leq 1.618$

$H_1: \mu > 1.618$

Suppose the true mean of B/A in the population is 1.65.

$\sigma = 0.148$

Give the sample size n required to reject H_0: $\mu \leq 1.618$ vs H_1: $\mu > 1.618$ with power ≥ 0.80. Use $S_n = 0.148$ as a guess of σ.

```
alpha <- 0.05
gm <- 0.80
sigma <- sd(gr)
mu <- 1.65
mu0 <- 1.618
za <- qnorm(1 - alpha)
zb <- qnorm(gm)
nr <- ceiling(sigma^2 * (za + zb)^2 / (mu - mu0)^2)
nr
```

```
[1] 133
```