STAT 516 Lec 10

Randomized complete block split-plot design

Karl Gregory

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Alfalfa data from Dr. Longnecker's notes

- Six fields; three plots in each field; four sub-plots in each plot.
- Each plot randomly assigned a type of alfalfa.
- Each sub-plot randomly assigned a cutting date.
- Response for each subplot is yield in tons/acre in the following year.

Variety	Date	Blocks 1	2	3	4	5	6	TrT Mean $\bar{Y}_{ij.}$
Ladak	None	2.17	1.88	1.62	2.34	1.58	1.66	1.8750
	S1	1.58	1.26	1.22	1.59	1.25	0.94	1.3067
	S20	2.29	1.60	1.67	1.91	1.39	1.12	1.6633
	O7	2.23	2.01	1.82	2.10	1.66	1.10	1.8200
Cossack	None	2.33	2.01	1.70	1.78	1.42	1.35	1.7650
	S1	1.38	1.30	1.85	1.09	1.13	1.06	1.3017
	S20	1.86	1.70	1.81	1.54	1.67	0.88	1.5767
	O7	2.27	1.81	2.01	1.40	1.31	1.06	1.6433
Ranger	None	1.75	1.95	2.13	1.78	1.31	1.30	1.7033
	S1	1.52	1.47	1.80	1.37	1.01	1.31	1.4133
	S20	1.55	1.61	1.82	1.56	1.23	1.13	1.4833
	O7	1.56	1.72	1.99	1.55	1.51	1.33	1.6100

```r
alfalfa <- data.frame(yield = c(2.17,1.88,1.62,2.34,1.58,1.66,
                                1.56,1.26,1.22,1.59,1.25,0.94,
                                2.29,1.60,1.67,1.91,1.39,1.12,
                                2.23,2.01,1.82,2.10,1.66,1.10,
                                2.33,2.01,1.70,1.78,1.42,1.35,
                                1.38,1.30,1.85,1.09,1.13,1.06,
                                1.86,1.70,1.81,1.54,1.67,0.88,
                                2.27,1.81,2.01,1.40,1.31,1.06,
                                1.75,1.95,2.13,1.78,1.31,1.30,
                                1.52,1.47,1.80,1.37,1.01,1.31,
                                1.55,1.61,1.82,1.56,1.23,1.13,
                                1.56,1.72,1.99,1.55,1.51,1.33),
                      variety = as.factor(c(rep("ladak",24),
                                            rep("cossack",24),
                                            rep("ranger",24))),
                      date = as.factor(rep(c(rep("none",6),
                                             rep("sep01",6),
                                             rep("sep20",6),
                                             rep("oct07",6)),
                                           3)),
                      field = as.factor(rep(1:6,12)))
```

```r
head(alfalfa,n = 26)
```

```
   yield variety  date field
1   2.17   ladak  none     1
2   1.88   ladak  none     2
3   1.62   ladak  none     3
4   2.34   ladak  none     4
5   1.58   ladak  none     5
6   1.66   ladak  none     6
7   1.56   ladak sep01     1
8   1.26   ladak sep01     2
9   1.22   ladak sep01     3
10  1.59   ladak sep01     4
11  1.25   ladak sep01     5
12  0.94   ladak sep01     6
13  2.29   ladak sep20     1
14  1.60   ladak sep20     2
15  1.67   ladak sep20     3
16  1.91   ladak sep20     4
17  1.39   ladak sep20     5
18  1.12   ladak sep20     6
19  2.23   ladak oct07     1
20  2.01   ladak oct07     2
21  1.82   ladak oct07     3
22  2.10   ladak oct07     4
23  1.66   ladak oct07     5
24  1.10   ladak oct07     6
25  2.33 cossack  none     1
26  2.01 cossack  none     2
```

Randomized complete block split plot design

- Each EU randomly assigned to one level of whole-plot factor.
- Each EU receives all levels of split-plot factor in random order.
- Groups of EUs over which this is replicated are treated as blocks.

Treatment effects model for RCB split-plot design

Assume

$$Y_{ijk} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij} + C_k + (\tau C)_{ik} + \varepsilon_{ijk},$$

for $i = 1, \ldots, a$, $j = 1, \ldots, b$ and $k = 1, \ldots, c$, where

- Y_{ijk} is the response of sub-plot j from whole plot i in block k.
- the τ_i are treatment effects for the whole-plot factor.
- the γ_j are treatment effects for the split-plot factor.
- the $(\tau\gamma)_{ij}$ are interaction effects between the factors.
- the C_k are independent Normal$(0, \sigma_C^2)$ block effects.
- the $(\tau C)_{ik}$ are indep Normal$(0, \sigma_{AC}^2)$ whole-plot$\times$block effects.
- the ε_{ijk} are independent Normal$(0, \sigma_\varepsilon^2)$ error terms.

Define the cell means as

$$\mu_{ij} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij}, \quad i = 1, \ldots, a, \quad j = 1, \ldots, b.$$

Goals in the RCB split-plot design

In the RCB split-plot model we will focus on how to:

1. Decompose the variability in the Y_{ijk} into its sources.
2. Test for significance of main effects and interaction.
3. Estimate the variance components σ_C^2, σ_{AC}^2, and σ_ε^2.
4. Make comparisons between treatment means.
5. Check whether the model assumptions are satisfied.

Sums of squares for the RCB split-plot design

SS	Symbol	Formula
Total	SS_{Tot}	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(Y_{ijk}-\bar{Y}_{...})^2$
A	SS_A	$bc\sum_{i=1}^{a}(\bar{Y}_{i..}-\bar{Y}_{...})^2$
C	SS_C	$ab\sum_{k=1}^{c}(\bar{Y}_{..k}-\bar{Y}_{...})^2$
AC	SS_{AC}	$b\sum_{i=1}^{a}\sum_{k=1}^{c}(\bar{Y}_{i.k}-(\bar{Y}_{i..}+\bar{Y}_{..k}-\bar{Y}_{...}))^2$
B	SS_B	$ac\sum_{j=1}^{b}(\bar{Y}_{.j.}-\bar{Y}_{...})^2$
AB	SS_{AB}	$c\sum_{i=1}^{a}\sum_{j=1}^{b}(Y_{ij.}-(\bar{Y}_{i..}+\bar{Y}_{.j.}-\bar{Y}_{...}))^2$
Error	SS_{Error}	$SS_{Tot}-(SS_A+SS_B+SS_{AB}+SS_C+SS_{AC})$

- We have $SS_{Tot}=SS_A+SS_B+SS_{AB}+SS_C+SS_{AC}$.

Expected mean squares in RCB split plot design

Source	Df	Expected mean square
A	$a-1$	$bc\theta^2_A + b\sigma^2_{AC} + \sigma^2_\varepsilon$
C	$c-1$	$ab\sigma^2_C + b\sigma^2_{AC} + \sigma^2_\varepsilon$
AC	$(a-1)(c-1)$	$b\sigma^2_{AC} + \sigma^2_\varepsilon$
B	$b-1$	$ac\theta^2_B + \sigma^2_\varepsilon$
AB	$(a-1)(b-1)$	$c\theta^2_{AB} + \sigma^2_\varepsilon$
Error	$a(b-1)(c-1)$	σ^2_ε

In the above

- $\theta^2_A = (a-1)^{-1}\sum_{i=1}^{a}(\bar{\mu}_{i.} - \bar{\mu}_{..})^2$
- $\theta^2_B = (b-1)^{-1}\sum_{j=1}^{b}(\bar{\mu}_{.j} - \bar{\mu}_{..})^2$
- $\theta^2_{AB} = [(a-1)(b-1)]^{-1}\sum_{i=1}^{a}\sum_{j=1}^{b}(\mu_{ij} - (\bar{\mu}_{i.} + \bar{\mu}_{.j} - \bar{\mu}_{..}))^2$

ANOVA table for RCB split-plot design

Source	Df	SS	MS	F value
A	$a-1$	$\mathrm{SS_A}$	$\mathrm{MS_A}$	$F_\mathrm{A} = \mathrm{MS_A} / \mathrm{MS_{AC}}$
C	$c-1$	$\mathrm{SS_C}$	$\mathrm{MS_C}$	$F_\mathrm{C} = \mathrm{MS_C} / \mathrm{MS_{AC}}$
AC	$(a-1)(c-1)$	$\mathrm{SS_{AC}}$	$\mathrm{MS_{AC}}$	$F_\mathrm{AC} = \mathrm{MS_{AC}} / \mathrm{MS_{Error}}$
B	$b-1$	$\mathrm{SS_B}$	$\mathrm{MS_B}$	$F_\mathrm{B} = \mathrm{MS_B} / \mathrm{MS_{Error}}$
AB	$(a-1)(b-1)$	$\mathrm{SS_{AB}}$	$\mathrm{MS_{AB}}$	$F_\mathrm{AB} = \mathrm{MS_{AB}} / \mathrm{MS_{Error}}$
Error	$a(b-1)(c-1)$	$\mathrm{SS_{Error}}$	$\mathrm{MS_{Error}}$	
Total	$abc-1$	$\mathrm{SS_{Tot}}$		

1. Reject H_0: $\bar{\mu}_{1.} = \cdots = \bar{\mu}_{a.}$ if $F_\mathrm{A} > F_{a-1,(a-1)(c-1),\alpha}$.
2. Reject H_0: $\sigma^2_C = 0$ if $F_\mathrm{C} > F_{c-1,(a-1)(c-1),\alpha}$.
3. Reject H_0: $\sigma^2_{AC} = 0$ if $F_\mathrm{AC} > F_{(a-1)(c-1),a(b-1)(c-1),\alpha}$.
4. Reject H_0: $\bar{\mu}_{.1} = \cdots = \bar{\mu}_{.b}$ if $F_\mathrm{B} > F_{b-1,a(b-1)(c-1),\alpha}$.
5. R. H_0: $\mu_{ij} = \bar{\mu}_{i.} + \bar{\mu}_{.j} - \bar{\mu}_{..}\ \forall ij$ if $F_\mathrm{AB} > F_{(a-1)(b-1),a(b-1)(c-1),\alpha}$.

Alfalfa data (cont)

`anova()` on `lm()` output gives wrong p-vals for `variety` and `field`.

```
lm_out <- lm(yield ~ variety + date + variety:date + field + field:variety,
             data = alfalfa)
anova(lm_out)
```

```
Analysis of Variance Table

Response: yield
              Df Sum Sq Mean Sq F value    Pr(>F)
variety        2 0.1753 0.08763  3.1198 0.0538447 .
date           3 1.9727 0.65758 23.4123 2.789e-09 ***
field          5 4.1388 0.82775 29.4710 3.798e-13 ***
variety:date   6 0.2147 0.03579  1.2742 0.2883071
variety:field 10 1.3574 0.13574  4.8330 0.0001022 ***
Residuals     45 1.2639 0.02809
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
y <- alfalfa$yield
y... <- predict(lm(yield ~ 1,data = alfalfa))
yi.. <- predict(lm(yield ~ variety,data = alfalfa))
y.j. <- predict(lm(yield ~ date,data = alfalfa))
y..k <- predict(lm(yield ~ field,data = alfalfa))
yij. <- predict(lm(yield ~ variety + date + variety:date,data = alfalfa))
yi.k <- predict(lm(yield ~ variety + field + variety:field,data = alfalfa))

SST <- sum((y - y...)^2)
SSA <- sum((yi.. - y...)^2)
SSC <- sum((y..k - y...)^2)
SSAC <- sum((yi.k - (yi.. + y..k - y...))^2)
SSB <- sum((y.j. - y...)^2)
SSAB <- sum((yij. - (yi.. + y.j. - y...))^2)
SSE <- SST - (SSA + SSC + SSAC + SSB + SSAB)
```

```
a <- 3
b <- 4
c <- 6

MSA <- SSA / (a-1)
MSC <- SSC / (c-1)
MSAC <- SSAC / ((c-1)*(a-1))
MSB <- SSB / (b-1)
MSAB <- SSAB / ((a-1)*(b-1))
MSE <- SSE / (a*(b-1)*(c-1))

FA_incorrect <- MSA / MSE
FC_incorrect <- MSC / MSE
FA <- MSA / MSAC
FC <- MSC / MSAC
FAC <- MSAC / MSE
FB <- MSB / MSE
FAB <- MSAB / MSE

pA_incorrect <- 1 - pf(FA_incorrect,a-1,a*(b-1)*(c-1))
pC_incorrect <- 1 - pf(FC_incorrect,c-1,a*(b-1)*(c-1))
pA <- 1 - pf(FA,a-1,(c-1)*(a-1))
pC <- 1 - pf(FC,c-1,(c-1)*(a-1))
pAC  <- 1 - pf(FAC,(c-1)*(a-1),a*(b-1)*(c-1))
pB <- 1 - pf(FB,b-1,a*(b-1)*(c-1))
pAB <- 1 - pf(FAB,(a-1)*(b-1),a*(b-1)*(c-1))
```

Correct ANOVA table:

Source	Df	SS	MS	F value	p value
A	2	0.175	0.088	0.646	0.5449
C	5	4.139	0.828	6.098	0.0076
AC	10	1.357	0.136	4.833	0.0001
B	3	1.973	0.658	23.412	0.0000
AB	6	0.215	0.036	1.274	0.2883
Error	45	1.264	0.028		
Total	71	9.123			

Correct p values for fixed effects with `lmerTest` package:

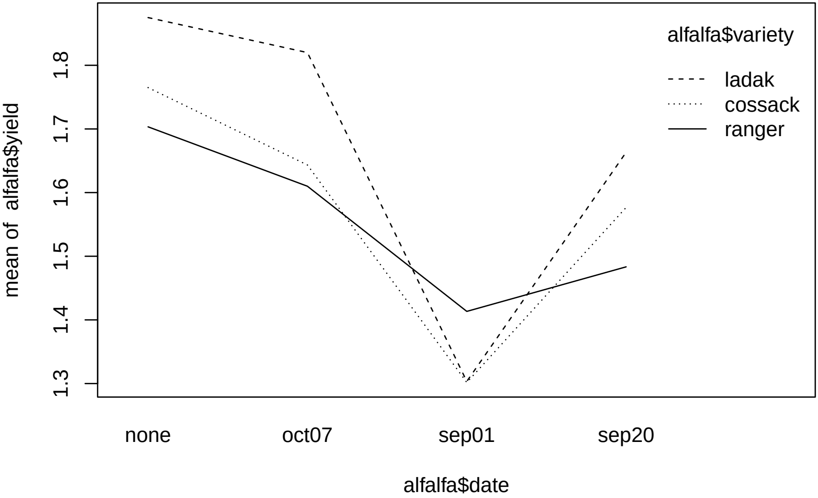
```
library(lmerTest) # first time run install.packages("lmerTest")
lmer_out <- lmer(yield ~ variety + date + variety:date + (1|field) + (1|field:variety),
                 data = alfalfa)
anova(lmer_out)
```

```
Type III Analysis of Variance Table with Satterthwaite's method
              Sum Sq Mean Sq NumDF DenDF F value    Pr(>F)
variety      0.03626 0.01813     2    10  0.6455    0.5449
date         1.97274 0.65758     3    45 23.4123 2.789e-09 ***
variety:date 0.21473 0.03579     6    45  1.2742    0.2883
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Rescales SS_{A} and MS_{A} by the factor $\mathrm{MS}_{\mathrm{Error}} \,/\, \mathrm{MS}_{\mathrm{AC}}$.

Note: If $\hat{\sigma}_C^2 = 0$, it will compute the whole-plot effect p value differently!

```r
interaction.plot(alfalfa$date,alfalfa$variety,alfalfa$yield)
```



MoMs for variance components in RCB split-plot design

The method of moments estimators for σ^2_C, σ^2_{AC}, and σ^2_ε are

- $\dot{\sigma}^2_\varepsilon = \mathrm{MS}_{\mathrm{Error}}$
- $\dot{\sigma}^2_{AC} = \dfrac{\mathrm{MS}_{\mathrm{AC}} - \mathrm{MS}_{\mathrm{Error}}}{b}$
- $\dot{\sigma}^2_C = \dfrac{\mathrm{MS}_{\mathrm{C}} - \mathrm{MS}_{\mathrm{AC}}}{ab}$

It is possible to obtain negative values for $\dot{\sigma}^2_{AC}$ and $\dot{\sigma}^2_C$. Use REML.

Alfalfa data (cont)

Obtain REML estimates of σ^2_C, σ^2_{AC}, and σ^2_ε from `lmer()`.

```
summary(lmer_out)$varcor
```

```
 Groups        Name        Std.Dev.
 field:variety (Intercept) 0.16406
 field         (Intercept) 0.24014
 Residual                  0.16759
```

Variances of some means and difference in means

Contrast	Variance	MoM variance estimator
$\bar{Y}_{i..} - \bar{Y}_{i'..}$	$\frac{2}{bc}(b\sigma^2_{AC} + \sigma^2_\varepsilon)$	$\frac{2}{bc}\,\mathrm{MS_{AC}}$
$\bar{Y}_{.j.} - \bar{Y}_{.j'.}$	$\frac{2}{ac}\sigma^2_\varepsilon$	$\frac{2}{ac}\,\mathrm{MS_{Error}}$
$\bar{Y}_{ij.} - \bar{Y}_{i'j.}$	$\frac{2}{c}(\sigma^2_{AC} + \sigma^2_\varepsilon)$	$\frac{2}{c}[\mathrm{MS_{AC}} + (b-1)\,\mathrm{MS_{Error}}]$
$\bar{Y}_{ij.} - \bar{Y}_{ij'.}$	$\frac{2}{c}\sigma^2_\varepsilon$	$\frac{2}{c}\,\mathrm{MS_{Error}}$
$\bar{Y}_{ij.} - \bar{Y}_{i'j'.}$	$\frac{2}{c}(\sigma^2_{AC} + \sigma^2_\varepsilon)$	$\frac{2}{c}[\mathrm{MS_{AC}} + (b-1)\,\mathrm{MS_{Error}}]$
$\bar{Y}_{i..}$	$\frac{1}{bc}[b\sigma^2_C + b\sigma^2_{AC} + \sigma^2_\varepsilon]$	$\frac{1}{abc}[\mathrm{MS_C} + (a-1)\,\mathrm{MS_{AC}}]$
$\bar{Y}_{.j.}$	$\frac{1}{bc}[a\sigma^2_C + \sigma^2_{AC} + \sigma^2_\varepsilon]$	$\frac{1}{abc}[\mathrm{MS_C} + (b-1)\,\mathrm{MS_{AC}}]$
$\bar{Y}_{ij.}$	$\frac{1}{c}[\sigma^2_C + \sigma^2_{AC} + \sigma^2_\varepsilon]$	$\frac{1}{abc}[\mathrm{MS_C} + (a-1)\,\mathrm{MS_{AC}} + a(b-1)\,\mathrm{MS_{Error}}]$

Some (unadjusted) CIs in RCB split plot design

Target	$(1-\alpha)100\%$ confidence interval
$\bar{\mu}_{i.} - \bar{\mu}_{i'.}$	$\bar{Y}_{i..} - \bar{Y}_{i'..} \pm t_{(a-1)(c-1),\alpha/2} \sqrt{\mathrm{MS}_{\mathrm{AC}}} \sqrt{\frac{2}{bc}}$
$\bar{\mu}_{.j} - \bar{\mu}_{.j'}$	$\bar{Y}_{.j.} - \bar{Y}_{.j'.} \pm t_{a(b-1)(c-1),\alpha/2} \sqrt{\mathrm{MS}_{\mathrm{Error}}} \sqrt{\frac{2}{ac}}$
$\mu_{ij} - \mu_{i'j}$	$\bar{Y}_{ij.} - \bar{Y}_{i'j.} \pm t_{\nu^*,\alpha/2} \sqrt{\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}} \sqrt{\frac{2}{c}}$
$\mu_{ij} - \mu_{ij'}$	$\bar{Y}_{ij.} - \bar{Y}_{ij'.} \pm t_{a(b-1)(c-1),\alpha/2} \sqrt{\mathrm{MS}_{\mathrm{Error}}} \sqrt{\frac{2}{c}}$
$\mu_{ij} - \mu_{i'j'}$	$\bar{Y}_{ij.} - \bar{Y}_{i'j'.} \pm t_{\nu^*,\alpha/2} \sqrt{\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}} \sqrt{\frac{2}{c}}$

In the above $\nu^* = \dfrac{\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}}{\frac{\mathrm{MS}_{\mathrm{AC}}^2}{(a-1)(c-1)} + \frac{(b-1)^2\,\mathrm{MS}_{\mathrm{Error}}^2}{a(b-1)(c-1)}}$ à la Satterthwaite[1].

[1] a degrees of freedom approximation when one has not exactly a t-distribution.

Alfalfa data (cont)

Dunnett's comparison of marginal date means with *no cutting* as baseline:

$$\bar{Y}_{.j.} - \bar{Y}_{.1.} \pm d_{b-1,a(b-1)(c-1),\alpha}\sqrt{\mathrm{MS}_{\mathrm{Error}}}\sqrt{\frac{2}{ac}}$$

Replace, in previous slide, $t_{a(b-1)(c-1),\alpha/2}$ with $d_{b-1,a(b-1)(c-1),\alpha}$.

In Dunnett's table we cannot find $d_{4,45,0.05}$, so take $d_{4,40,0.05} = 2.44$.

Table A.5 Critical Values for Dunnett's Two-Sided Test of Treatments versus Control.

		T = Number of Groups Counting Both Treatments and Control						
Error df	**Two-sided α**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
5	0.05	2.57	3.03	3.29	3.48	3.62	3.73	3.82
5	0.01	4.03	4.63	4.97	5.22	5.41	5.56	5.68
6	0.05	2.45	2.86	3.10	3.26	3.39	3.49	3.57
6	0.01	3.71	4.21	4.51	4.71	4.87	5.00	5.10
7	0.05	2.36	2.75	2.97	3.12	3.24	3.33	3.41
7	0.01	3.50	3.95	4.21	4.39	4.53	4.64	4.74
8	0.05	2.31	2.67	2.88	3.02	3.13	3.22	3.29
8	0.01	3.36	3.77	4.00	4.17	4.29	4.40	4.48
9	0.05	2.26	2.61	2.81	2.95	3.05	3.14	3.20
9	0.01	3.25	3.63	3.85	4.01	4.12	4.22	4.30
10	0.05	2.23	2.57	2.76	2.89	2.99	3.07	3.14
10	0.01	3.17	3.53	3.74	3.88	3.99	4.08	4.16
11	0.05	2.20	2.53	2.72	2.84	2.94	3.02	3.08
11	0.01	3.11	3.45	3.65	3.79	3.89	3.98	4.05
12	0.05	2.18	2.50	2.68	2.81	2.90	2.98	3.04
12	0.01	3.05	3.39	3.58	3.71	3.81	3.89	3.96
13	0.05	2.16	2.48	2.65	2.78	2.87	2.94	3.00
13	0.01	3.01	3.33	3.52	3.65	3.74	3.82	3.89
14	0.05	2.14	2.46	2.63	2.75	2.84	2.91	2.97
14	0.01	2.98	3.29	3.47	3.59	3.69	3.76	3.83
15	0.05	2.13	2.44	2.61	2.73	2.82	2.89	2.95
15	0.01	2.95	3.25	3.43	3.55	3.64	3.71	3.78
16	0.05	2.12	2.42	2.59	2.71	2.80	2.87	2.92
16	0.01	2.92	3.22	3.39	3.51	3.60	3.67	3.73
17	0.05	2.11	2.41	2.58	2.69	2.78	2.85	2.90
17	0.01	2.90	3.19	3.36	3.47	3.56	3.63	3.69
18	0.05	2.10	2.40	2.56	2.68	2.76	2.83	2.89
18	0.01	2.88	3.17	3.33	3.44	3.53	3.60	3.66
19	0.05	2.09	2.39	2.55	2.66	2.75	2.81	2.87
19	0.01	2.86	3.15	3.31	3.42	3.50	3.57	3.63
20	0.05	2.09	2.38	2.54	2.65	2.73	2.80	2.86
20	0.01	2.85	3.13	3.29	3.40	3.48	3.55	3.60
25	0.05	2.06	2.34	2.50	2.61	2.69	2.75	2.81
25	0.01	2.79	3.06	3.21	3.31	3.39	3.45	3.51
30	0.05	2.04	2.32	2.47	2.58	2.66	2.72	2.77
30	0.01	2.75	3.01	3.15	3.25	3.33	3.39	3.44
40	0.05	2.02	2.29	2.44	2.54	2.62	2.68	2.73
40	0.01	2.70	2.95	3.09	3.19	3.26	3.32	3.37
60	0.05	2.00	2.27	2.41	2.51	2.58	2.64	2.69
60	0.01	2.66	2.90	3.03	3.12	3.19	3.25	3.29

This table produced from the SAS System using function PROBMC('DUNNETT2',,1 − α,df,k), where $k = T - 1$.

Figure 1: Table A.5 from Mohr, Wilson, and Freund (2021)

```r
y.1.bar <- mean(alfalfa$yield[alfalfa$date=="none"])
y.2.bar <- mean(alfalfa$yield[alfalfa$date=="sep01"])
y.3.bar <- mean(alfalfa$yield[alfalfa$date=="sep20"])
y.4.bar <- mean(alfalfa$yield[alfalfa$date=="oct07"])


se <- sqrt(MSE) * sqrt(2/(a*c))
me <- 2.44 * se

dtab <- rbind(c(y.2.bar - y.1.bar - me,y.2.bar - y.1.bar + me),
              c(y.3.bar - y.1.bar - me,y.3.bar - y.1.bar + me),
              c(y.4.bar - y.1.bar - me,y.4.bar - y.1.bar + me))

colnames(dtab) <- c("lower","upper")
rownames(dtab) <- c("sep01 - none",
                    "sep20 - none",
                    "oct07 - none")
round(dtab,3)
```

```
             lower  upper
sep01 - none -0.578 -0.305
sep20 - none -0.343 -0.070
oct07 - none -0.226  0.046
```

Unadjusted CIs with `ls_means()` from R package `lmerTest`:

```
ls_means(lmer_out,which="date")
```

```
Least Squares Means table:

          Estimate Std. Error  df t value   lower   upper  Pr(>|t|)
datenone   1.78111    0.11255 6.1  15.825 1.50641 2.05581 3.684e-06 ***
dateoct07  1.69111    0.11255 6.1  15.026 1.41641 1.96581 5.010e-06 ***
datesep01  1.33944    0.11255 6.1  11.901 1.06474 1.61415 1.977e-05 ***
datesep20  1.57444    0.11255 6.1  13.989 1.29974 1.84915 7.646e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

  Confidence level: 95%
  Degrees of freedom method: Satterthwaite
```

```r
ls_means(lmer_out, pairwise = TRUE, which = "date")
```

```
Least Squares Means table:

                          Estimate Std. Error df t value      lower      upper
datenone - dateoct07   0.0900000  0.0558639 45  1.6111 -0.0225156  0.2025156
datenone - datesep01   0.4416667  0.0558639 45  7.9061  0.3291511  0.5541823
datenone - datesep20   0.2066667  0.0558639 45  3.6995  0.0941511  0.3191823
dateoct07 - datesep01  0.3516667  0.0558639 45  6.2951  0.2391511  0.4641823
dateoct07 - datesep20  0.1166667  0.0558639 45  2.0884  0.0041511  0.2291823
datesep01 - datesep20 -0.2350000  0.0558639 45 -4.2067 -0.3475156 -0.1224844
                        Pr(>|t|)
datenone - dateoct07  0.1141598
datenone - datesep01  4.723e-10 ***
datenone - datesep20  0.0005859 ***
dateoct07 - datesep01 1.138e-07 ***
dateoct07 - datesep20 0.0424494 *
datesep01 - datesep20 0.0001219 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

  Confidence level: 95%
  Degrees of freedom method: Satterthwaite
```

References

Mohr, Donna L, William J Wilson, and Rudolf J Freund. 2021. *Statistical Methods*. Academic Press.