STAT 516 Lec 10

Randomized complete block split-plot design

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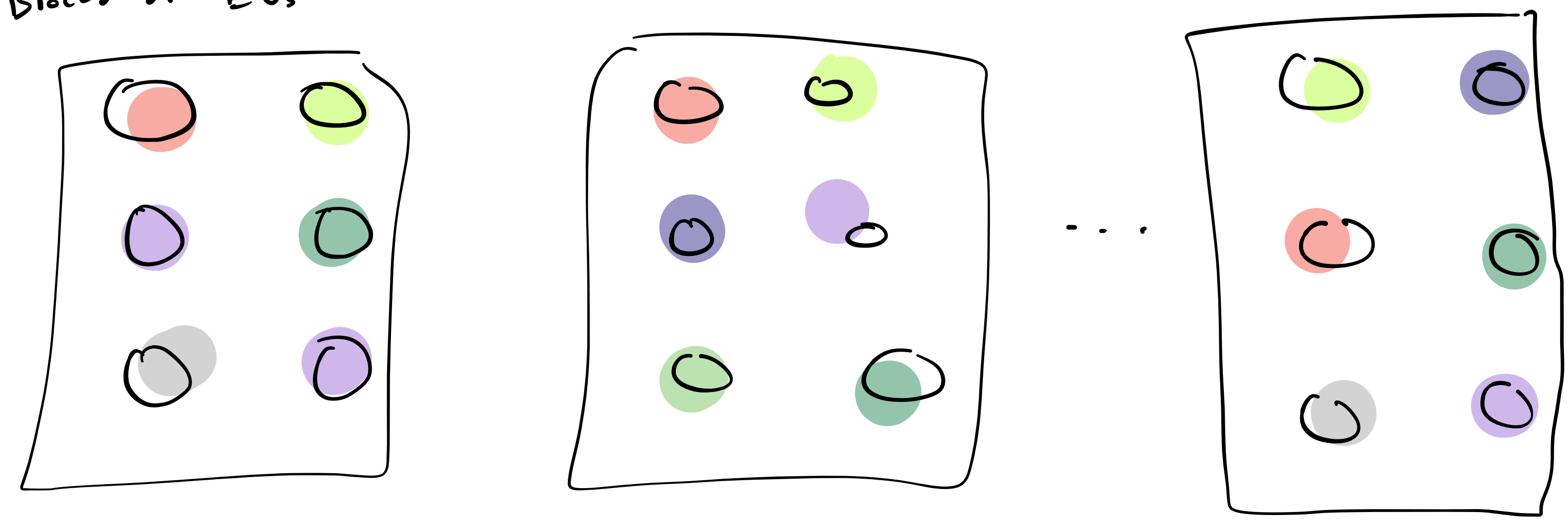
2025-04-15

Randomized complete block design (RCBD)

* Group EUs

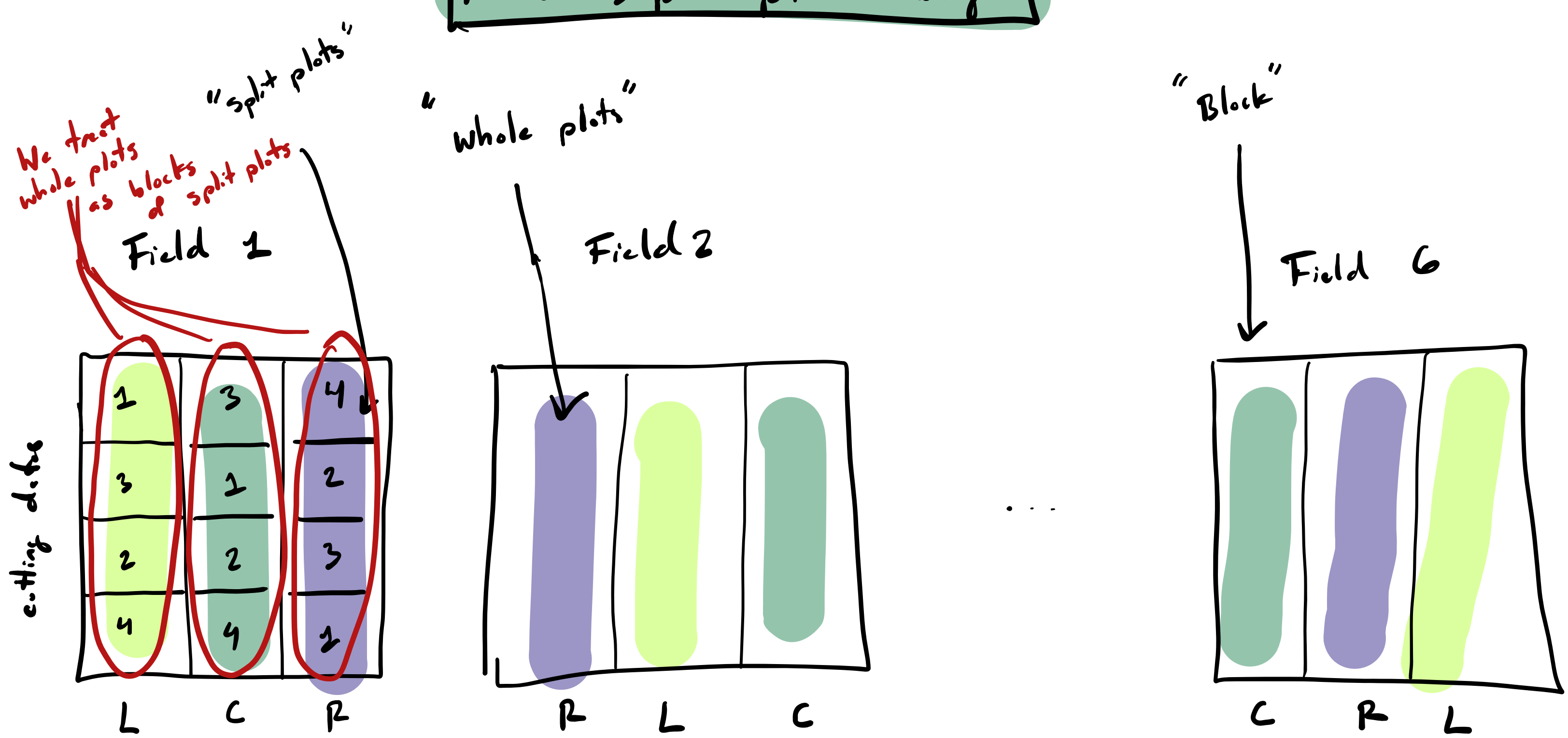
Blocks of EUs



2 Factors A and B. Within each block, assign randomly all combinations of factor levels

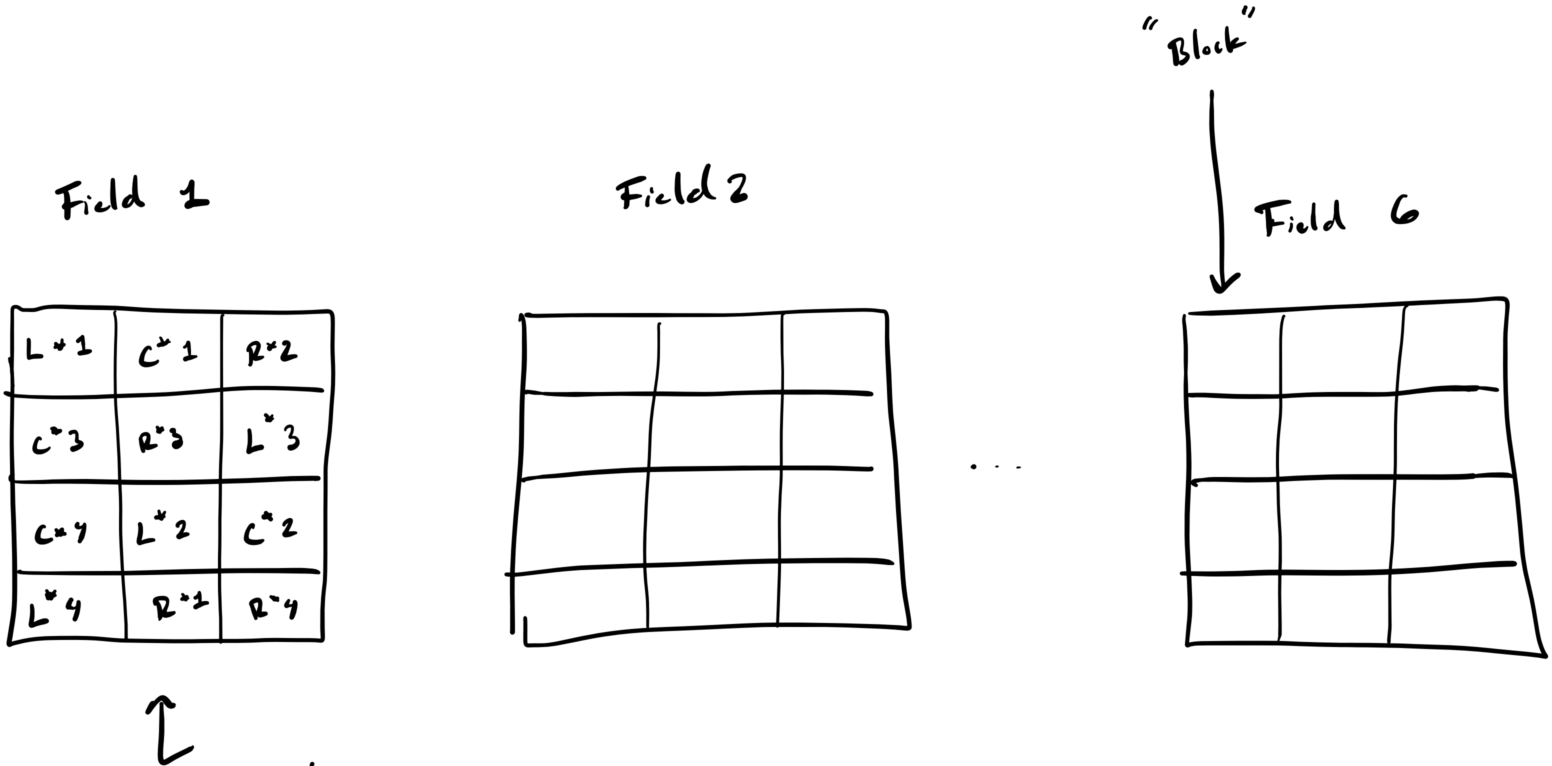
If $a = 3$, $b = 2$ → 6 total factor level combos.

RCB Split-plot design



RCBD would look like this:

$a=3$, $b=4$, so $ab=12$ total treatment combinations



Randomly assign each of the 12 smaller plots to one of the 12 treatment level combinations.

Alfalfa data from Dr. Longnecker's notes

- Six fields; three plots in each field; four sub-plots in each plot.
- Each plot randomly assigned a type of alfalfa.
- Each sub-plot randomly assigned a cutting date.
- Response for each subplot is yield in $\boxed{\text{tons/acre}}$ in the following year.

Variety	Date	Blocks 1	2	3	4	5	6	TrT Mean $\bar{Y}_{ij.}$
Ladak	None	2.17	1.88	1.62	2.34	1.58	1.66	1.8750
Sep 1	S1	1.58	1.26	1.22	1.59	1.25	0.94	1.3067
Sep 20	S20	2.29	1.60	1.67	1.91	1.39	1.12	1.6633
Oct 7	O7	2.23	2.01	1.82	2.10	1.66	1.10	1.8200
Cossack	None	2.33	2.01	1.70	1.78	1.42	1.35	1.7650
	S1	1.38	1.30	1.85	1.09	1.13	1.06	1.3017
	S20	1.86	1.70	1.81	1.54	1.67	0.88	1.5767
	O7	2.27	1.81	2.01	1.40	1.31	1.06	1.6433
Ranger	None	1.75	1.95	2.13	1.78	1.31	1.30	1.7033
	S1	1.52	1.47	1.80	1.37	1.01	1.31	1.4133
	S20	1.55	1.61	1.82	1.56	1.23	1.13	1.4833
	O7	1.56	1.72	1.99	1.55	1.51	1.33	1.6100

```r
alfalfa <- data.frame(yield = c(2.17,1.88,1.62,2.34,1.58,1.66,
                                1.56,1.26,1.22,1.59,1.25,0.94,
                                2.29,1.60,1.67,1.91,1.39,1.12,
                                2.23,2.01,1.82,2.10,1.66,1.10,
                                2.33,2.01,1.70,1.78,1.42,1.35,
                                1.38,1.30,1.85,1.09,1.13,1.06,
                                1.86,1.70,1.81,1.54,1.67,0.88,
                                2.27,1.81,2.01,1.40,1.31,1.06,
                                1.75,1.95,2.13,1.78,1.31,1.30,
                                1.52,1.47,1.80,1.37,1.01,1.31,
                                1.55,1.61,1.82,1.56,1.23,1.13,
                                1.56,1.72,1.99,1.55,1.51,1.33),
                      variety = as.factor(c(rep("ladak",24),
                                            rep("cossack",24),
                                            rep("ranger",24))),
                      date = as.factor(rep(c(rep("none",6),
                                             rep("sep01",6),
                                             rep("sep20",6),
                                             rep("oct07",6)),
                                           3)),
                      field = as.factor(rep(1:6,12)))
```

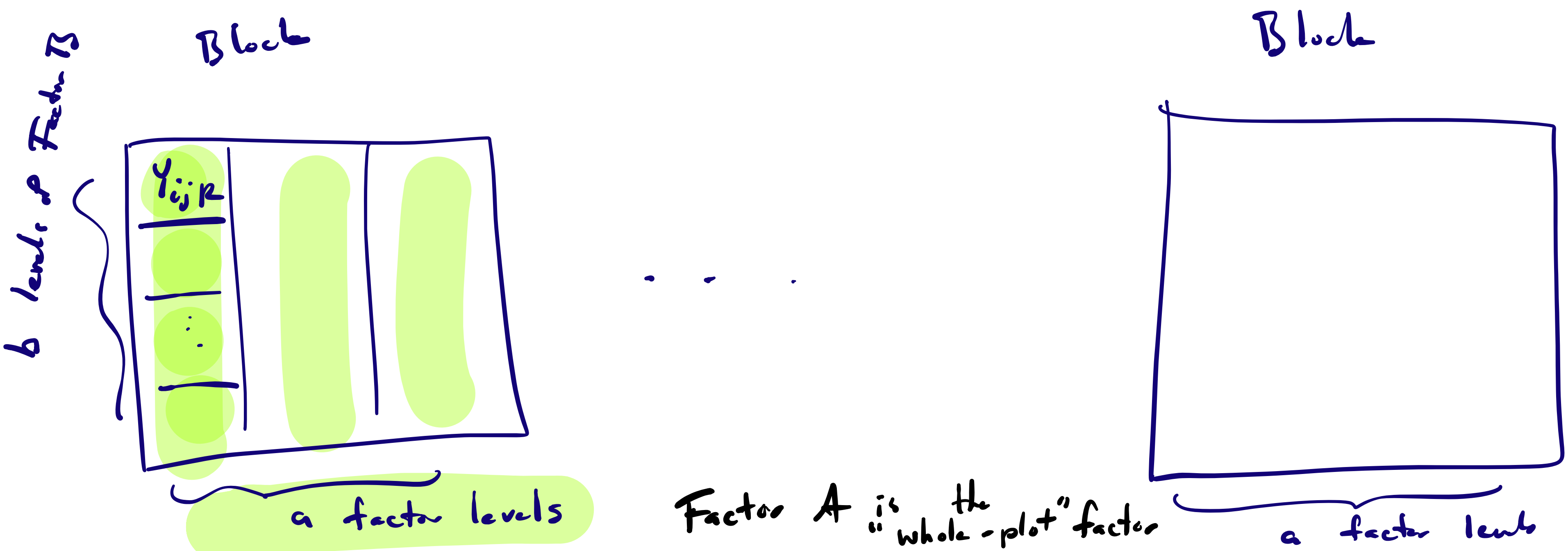
```r
head(alfalfa,n = 26)
```

```
   yield variety  date field
1   2.17   ladak  none     1
2   1.88   ladak  none     2
3   1.62   ladak  none     3
4   2.34   ladak  none     4
5   1.58   ladak  none     5
6   1.66   ladak  none     6
7   1.56   ladak sep01     1
8   1.26   ladak sep01     2
9   1.22   ladak sep01     3
10  1.59   ladak sep01     4
11  1.25   ladak sep01     5
12  0.94   ladak sep01     6
13  2.29   ladak sep20     1
14  1.60   ladak sep20     2
15  1.67   ladak sep20     3
16  1.91   ladak sep20     4
17  1.39   ladak sep20     5
18  1.12   ladak sep20     6
19  2.23   ladak oct07     1
20  2.01   ladak oct07     2
21  1.82   ladak oct07     3
22  2.10   ladak oct07     4
23  1.66   ladak oct07     5
24  1.10   ladak oct07     6
25  2.33 cossack  none     1
26  2.01 cossack  none     2
```

Randomized complete block split plot design

- Each EU randomly assigned to one level of whole-plot factor.
- Each EU receives all levels of split-plot factor in random order.
- Groups of EUs over which this is replicated are treated as blocks.

Factor B is the split-plot factor



Treatment effects model for RCB split-plot design

Not in the RCBD.

Assume

k = block
i = level of factor A
j = level of factor B

μ_{ij} — block effect — captures homogeneity in the subplots within each whole plot.

$$Y_{ijk} = \boxed{\mu + \tau_i + \gamma_j + (\tau\gamma)_{ij}} + C_k + (\tau C)_{ik} + \varepsilon_{ijk},$$

baseline mean — a random effect

for $i = 1, \ldots, a$, $j = 1, \ldots, b$ and $k = 1, \ldots, c$, where

- Y_{ijk} is the response of sub-plot j from whole plot i in block k.
- the τ_i are treatment effects for the whole-plot factor. (A)
- the γ_j are treatment effects for the split-plot factor. (B)
- the $(\tau\gamma)_{ij}$ are interaction effects between the factors. (A×B)
- the C_k are independent Normal$(0, \sigma_C^2)$ block effects.
- the $(\tau C)_{ik}$ are indep Normal$(0, \sigma_{AC}^2)$ whole-plot×block effects.
- the ε_{ijk} are independent Normal$(0, \sigma_\varepsilon^2)$ error terms.

Define the cell means as

$$\boxed{\mu_{ij} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij},} \quad i = 1, \ldots, a, \quad j = 1, \ldots, b.$$

Goals in the RCB split-plot design

In the RCB split-plot model we will focus on how to:

1. Decompose the variability in the Y_{ijk} into its sources.
2. Test for significance of main effects and interaction.
3. Estimate the variance components σ^2_C, σ^2_{AC}, and σ^2_ε.
4. Make comparisons between treatment means.
5. Check whether the model assumptions are satisfied.

Sums of squares for the RCB split-plot design

	SS		Symbol	Formula
	Total		$\mathrm{SS_{Tot}}$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(Y_{ijk}-\bar{Y}_{...})^2$
τ_i:	A	whole-plot factor SS	$\boxed{\mathrm{SS_A}}$	$bc\sum_{i=1}^{a}(\bar{Y}_{i..}-\bar{Y}_{...})^2$
c_k	C	Block	$\mathrm{SS_C}$	$ab\sum_{k=1}^{c}(\bar{Y}_{..k}-\bar{Y}_{...})^2$
$(\tau c)_{ik}$	AC	whole-plot × Block interaction	$\mathrm{SS_{AC}}$	$b\sum_{i=1}^{a}\sum_{k=1}^{c}(\bar{Y}_{i.k}-(\bar{Y}_{i..}+\bar{Y}_{..k}-\bar{Y}_{...}))^2$
γ_j	B	split-plot	$\mathrm{SS_B}$	$ac\sum_{j=1}^{b}(\bar{Y}_{.j.}-\bar{Y}_{...})^2$
$(\tau\gamma)_{ij}$	AB	A×B interaction	$\mathrm{SS_{AB}}$	$c\sum_{i=1}^{a}\sum_{j=1}^{b}(Y_{ij.}-(\bar{Y}_{i..}+\bar{Y}_{.j.}-\bar{Y}_{...}))^2$
ε_{ijk}	Error		$\mathrm{SS_{Error}}$	$\mathrm{SS_{Tot}}-(\mathrm{SS_A}+\mathrm{SS_B}+\mathrm{SS_{AB}}+\mathrm{SS_C}+\mathrm{SS_{AC}})$

- We have $\mathrm{SS_{Tot}} = \mathrm{SS_A}+\mathrm{SS_B}+\mathrm{SS_{AB}}+\mathrm{SS_C}+\mathrm{SS_{AC}}$.

Expected mean squares in RCB split plot design

Under $H_0: \bar{\mu}_{1.} = \dots = \bar{\mu}_{a.}$, this disappears.

$c = \#$ blocks

$MS_A = \frac{SSA}{a-1}$

$MS_C = \frac{SSC}{c-1}$

Source	Df	Expected mean square	
A	$a-1$	$\cancel{bc\theta^2_A} + b\sigma^2_{AC} + \sigma^2_\varepsilon$	$\mathbb{E}\, MS_A$
C	$c-1$	$ab\sigma^2_C + b\sigma^2_{AC} + \sigma^2_\varepsilon$	$\mathbb{E}\, MS_C$
AC	$(a-1)(c-1)$	$b\sigma^2_{AC} + \sigma^2_\varepsilon$	$\mathbb{E}\, MS_{AC}$
B	$b-1$	$ac\theta^2_B + \sigma^2_\varepsilon$	
AB	$(a-1)(b-1)$	$c\theta^2_{AB} + \sigma^2_\varepsilon$	
Error	$a(b-1)(c-1)$	σ^2_ε	$\mathbb{E}\, MS_{Error}$

same, so use $F_A = MSA / MS_{AC}$

Consider: $H_0: \bar{\mu}_{1.} = \dots = \bar{\mu}_{a.}$ (No Factor A main effect.)

In the above

- $\boxed{\theta^2_A = (a-1)^{-1} \sum_{i=1}^{a} (\bar{\mu}_{i.} - \bar{\mu}_{..})^2}$ ← Measures variation in the marginal means of factor A.
- $\theta^2_B = (b-1)^{-1} \sum_{j=1}^{b} (\bar{\mu}_{.j} - \bar{\mu}_{..})^2$
- $\theta^2_{AB} = [(a-1)(b-1)]^{-1} \sum_{i=1}^{a} \sum_{j=1}^{b} (\mu_{ij} - (\bar{\mu}_{i.} + \bar{\mu}_{.j} - \bar{\mu}_{..}))^2$

ANOVA table for RCB split-plot design

	Source	Df	SS	MS	F value
τ_i	A	$a-1$	$\mathrm{SS_A}$	$\mathrm{MS_A}$	$F_\mathrm{A} = \mathrm{MS_A} \boxed{/\,\mathrm{MS_{AC}}}$
c_k	C	$c-1$	$\mathrm{SS_C}$	$\mathrm{MS_C}$	$F_\mathrm{C} = \mathrm{MS_C} \boxed{/\,\mathrm{MS_{AC}}}$
$(\tau c)_{ik}$	AC	$(a-1)(c-1)$	$\mathrm{SS_{AC}}$	$\mathrm{MS_{AC}}$	$F_\mathrm{AC} = \mathrm{MS_{AC}}\,/\,\mathrm{MS_{Error}}$
β_j	B	$b-1$	$\mathrm{SS_B}$	$\mathrm{MS_B}$	$F_\mathrm{B} = \mathrm{MS_B}\,/\,\mathrm{MS_{Error}}$
$(\tau\beta)_{ij}$	AB	$(a-1)(b-1)$	$\mathrm{SS_{AB}}$	$\mathrm{MS_{AB}}$	$F_\mathrm{AB} = \mathrm{MS_{AB}}\,/\,\mathrm{MS_{Error}}$
ε_{ijk}	Error	$a(b-1)(c-1)$	$\mathrm{SS_{Error}}$	$\mathrm{MS_{Error}}$	
	Total	$abc-1$	$\mathrm{SS_{Tot}}$		

If $\sigma^2_{AC} = 0$, we could just do the RCBD.

"No factor A main effect."

1. Reject H_0: $\bar{\mu}_{1.} = \cdots = \bar{\mu}_{a.}$ if $F_\mathrm{A} > F_{a-1,(a-1)(c-1),\alpha}$.
2. Reject H_0: $\sigma^2_C = 0$ if $F_\mathrm{C} > F_{c-1,(a-1)(c-1),\alpha}$. H_0: No block effect
3. Reject H_0: $\sigma^2_{AC} = 0$ if $F_\mathrm{AC} > F_{(a-1)(c-1),a(b-1)(c-1),\alpha}$. H_0: No effect of the whole plot within each block
4. Reject H_0: $\bar{\mu}_{.1} = \cdots = \bar{\mu}_{.b}$ if $F_\mathrm{B} > F_{b-1,a(b-1)(c-1),\alpha}$.
5. R. H_0: $\mu_{ij} = \bar{\mu}_{i.} + \bar{\mu}_{.j} - \bar{\mu}_{..}\ \forall ij$ if $F_\mathrm{AB} > F_{(a-1)(b-1),a(b-1)(c-1),\alpha}$.

Alfalfa data (cont)

anova() on lm() output gives wrong p-vals for variety and field.

```
lm_out <- lm(yield ~ variety + date + variety:date + field + field:variety,
             data = alfalfa)
anova(lm_out)
```

```
Analysis of Variance Table

Response: yield
              Df Sum Sq Mean Sq F value    Pr(>F)
variety        2 0.1753 0.08763  3.1198 0.0538447 .
date           3 1.9727 0.65758 23.4123 2.789e-09 ***
field          5 4.1388 0.82775 29.4710 3.798e-13 ***
variety:date   6 0.2147 0.03579  1.2742 0.2883071
variety:field 10 1.3574 0.13574  4.8330 0.0001022 ***
Residuals     45 1.2639 0.02809
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

B (date)

AB (variety:date)

AC (variety:field)

Incorrect

H_0: No cutting date effect.

Incorrect

H_0: No interaction

p-value for testing $H_0: \sigma^2_{AC} = 0$.

The p-value is small so we reject H_0.

Whole-plot X block interaction

So we should stick with the split-plot design.

```r
y <- alfalfa$yield
y... <- predict(lm(yield ~ 1,data = alfalfa))
yi.. <- predict(lm(yield ~ variety,data = alfalfa))
y.j. <- predict(lm(yield ~ date,data = alfalfa))
y..k <- predict(lm(yield ~ field,data = alfalfa))
yij. <- predict(lm(yield ~ variety + date + variety:date,data = alfalfa))
yi.k <- predict(lm(yield ~ variety + field + variety:field,data = alfalfa))

SST <- sum((y - y...)^2)
SSA <- sum((yi.. - y...)^2)
SSC <- sum((y..k - y...)^2)
SSAC <- sum((yi.k - (yi.. + y..k - y...))^2)
SSB <- sum((y.j. - y...)^2)
SSAB <- sum((yij. - (yi.. + y.j. - y...))^2)
SSE <- SST - (SSA + SSC + SSAC + SSB + SSAB)
```

```r
a <- 3
b <- 4
c <- 6

MSA <- SSA / (a-1)
MSC <- SSC / (c-1)
MSAC <- SSAC / ((c-1)*(a-1))
MSB <- SSB / (b-1)
MSAB <- SSAB / ((a-1)*(b-1))
MSE <- SSE / (a*(b-1)*(c-1))

FA_incorrect <- MSA / MSE
FC_incorrect <- MSC / MSE
FA <- MSA / MSAC
FC <- MSC / MSAC
FAC <- MSAC / MSE
FB <- MSB / MSE
FAB <- MSAB / MSE

pA_incorrect <- 1 - pf(FA_incorrect,a-1,a*(b-1)*(c-1))
pC_incorrect <- 1 - pf(FC_incorrect,c-1,a*(b-1)*(c-1))
pA <- 1 - pf(FA,a-1,(c-1)*(a-1))
pC <- 1 - pf(FC,c-1,(c-1)*(a-1))
pAC  <- 1 - pf(FAC,(c-1)*(a-1),a*(b-1)*(c-1))
pB <- 1 - pf(FB,b-1,a*(b-1)*(c-1))
pAB <- 1 - pf(FAB,(a-1)*(b-1),a*(b-1)*(c-1))
```

Correct p-values

Correct ANOVA table:

Source	Df	SS	MS	F value	p value
A	2	0.175	0.088	0.646	0.5449
C	5	4.139	0.828	6.098	0.0076
AC	10	1.357	0.136	4.833	0.0001
B	3	1.973	0.658	23.412	0.0000
AB	6	0.215	0.036	1.274	0.2883
Error	45	1.264	0.028		
Total	71	9.123			

H_0: no alfalfa effect

H_0: no field effect.

H_0: no cutting date effect

H_0: no variety & date interaction

Correct p values for fixed effects with `lmerTest` package:

```
library(lmerTest) # first time run install.packages("lmerTest")
lmer_out <- lmer(yield ~ variety + date + variety:date + (1|field) + (1|field:variety),
                 data = alfalfa)
anova(lmer_out)
```

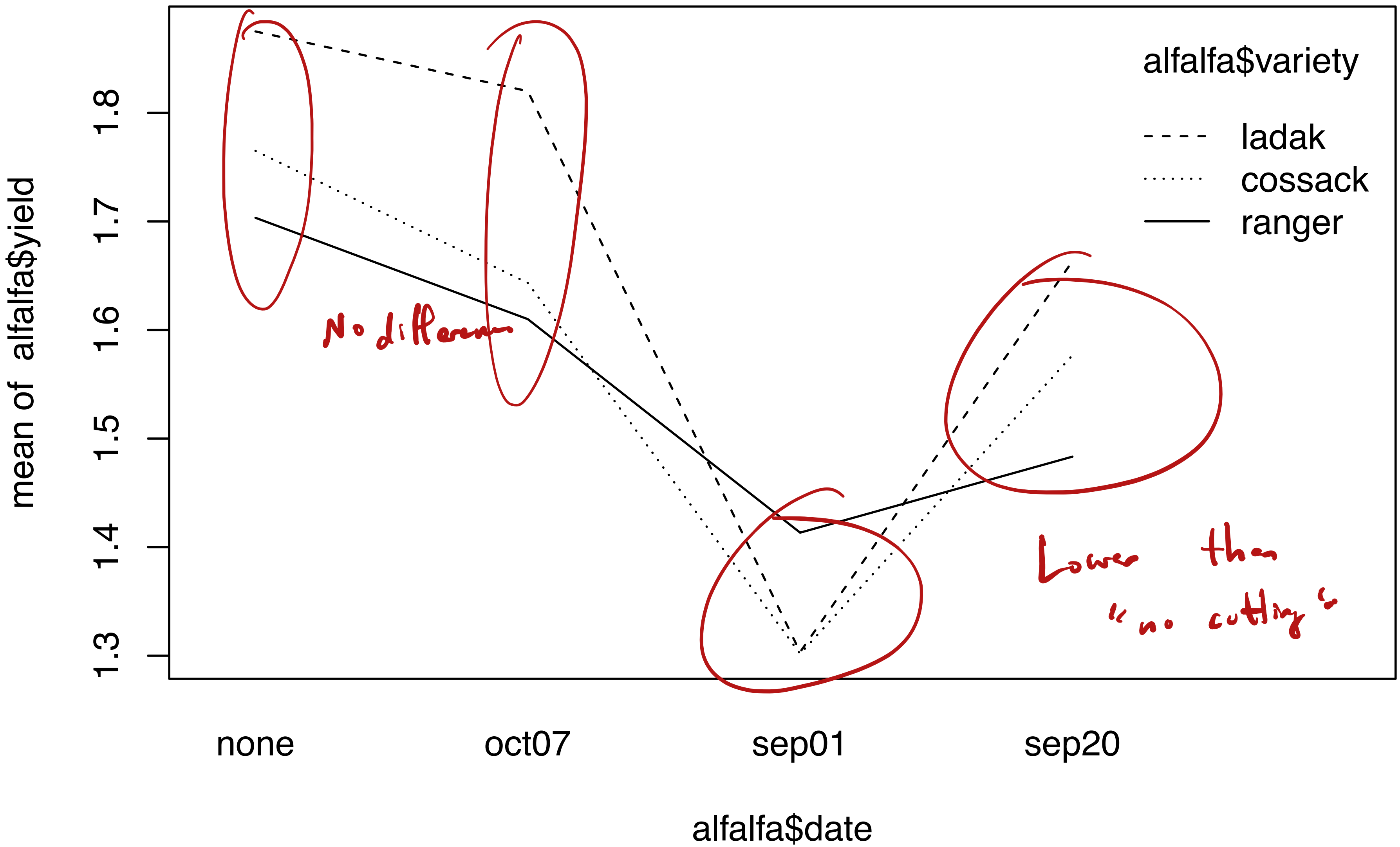
```
Type III Analysis of Variance Table with Satterthwaite's method
               Sum Sq Mean Sq NumDF DenDF F value    Pr(>F)
A  variety      0.03626 0.01813     2    10  0.6455    0.5449
B  date         1.97274 0.65758     3    45 23.4123 2.789e-09 ***
AB variety:date 0.21473 0.03579     6    45  1.2742    0.2883
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Correct p-values!

Rescales SS_A and MS_A by the factor $\mathrm{MS}_\mathrm{Error} \,/\, \mathrm{MS}_\mathrm{AC}$.

Note: If $\hat{\sigma}^2_C = 0$, it will compute the whole-plot effect p value differently!

```r
interaction.plot(alfalfa$date,alfalfa$variety,alfalfa$yield)
```



MoMs for variance components in RCB split-plot design

The method of moments estimators for σ_C^2, σ_{AC}^2, and σ_ε^2 are

- $\dot{\sigma}_\varepsilon^2 = \mathrm{MS_{Error}}$
- $\dot{\sigma}_{AC}^2 = \dfrac{\mathrm{MS_{AC}} - \mathrm{MS_{Error}}}{b}$
- $\dot{\sigma}_C^2 = \dfrac{\mathrm{MS_C} - \mathrm{MS_{AC}}}{ab}$

It is possible to obtain negative values for $\dot{\sigma}_{AC}^2$ and $\dot{\sigma}_C^2$. Use REML.

Alfalfa data (cont)

Obtain REML estimates of σ_C^2, σ_{AC}^2, and σ_ε^2 from `lmer()`.

```
summary(lmer_out)$varcor
```

```
Groups         Name        Std.Dev.
field:variety  (Intercept) 0.16406
field          (Intercept) 0.24014
Residual                   0.16759
```

$\hat{\sigma}_{AC}$

$\hat{\sigma}_{C}$

$\hat{\sigma}_{\varepsilon}$

Variances of some means and difference in means

Contrast	Variance	MoM variance estimator
$\bar{Y}_{i..} - \bar{Y}_{i'..}$	$\frac{2}{bc}(b\sigma^2_{AC} + \sigma^2_{\varepsilon})$	$\frac{2}{bc}\,\mathrm{MS}_{\mathrm{AC}}$
$\bar{Y}_{.j.} - \bar{Y}_{.j'.}$	$\frac{2}{ac}\sigma^2_{\varepsilon}$	$\frac{2}{ac}\,\mathrm{MS}_{\mathrm{Error}}$
$\bar{Y}_{ij.} - \bar{Y}_{i'j.}$	$\frac{2}{c}(\sigma^2_{AC} + \sigma^2_{\varepsilon})$	$\frac{2}{c}[\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}]$
$\bar{Y}_{ij.} - \bar{Y}_{ij'.}$	$\frac{2}{c}\sigma^2_{\varepsilon}$	$\frac{2}{c}\,\mathrm{MS}_{\mathrm{Error}}$
$\bar{Y}_{ij.} - \bar{Y}_{i'j'.}$	$\frac{2}{c}(\sigma^2_{AC} + \sigma^2_{\varepsilon})$	$\frac{2}{c}[\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}]$
$\bar{Y}_{i..}$	$\frac{1}{bc}[b\sigma^2_{C} + b\sigma^2_{AC} + \sigma^2_{\varepsilon}]$	$\frac{1}{abc}[\mathrm{MS}_{\mathrm{C}} + (a-1)\,\mathrm{MS}_{\mathrm{AC}}]$
$\bar{Y}_{.j.}$	$\frac{1}{bc}[a\sigma^2_{C} + \sigma^2_{AC} + \sigma^2_{\varepsilon}]$	$\frac{1}{abc}[\mathrm{MS}_{\mathrm{C}} + (b-1)\,\mathrm{MS}_{\mathrm{AC}}]$
$\bar{Y}_{ij.}$	$\frac{1}{c}[\sigma^2_{C} + \sigma^2_{AC} + \sigma^2_{\varepsilon}]$	$\frac{1}{abc}[\mathrm{MS}_{\mathrm{C}} + (a-1)\,\mathrm{MS}_{\mathrm{AC}} + a(b-1)\,\mathrm{MS}_{\mathrm{Error}}]$

Some (unadjusted) CIs in RCB split plot design

Compare two Factor B marginal means

Target	$(1-\alpha)100\%$ confidence interval
$\bar{\mu}_{i.} - \bar{\mu}_{i'.}$	$\bar{Y}_{i..} - \bar{Y}_{i'..} \pm t_{(a-1)(c-1),\alpha/2} \sqrt{\mathrm{MS}_{\mathrm{AC}}} \sqrt{\frac{2}{bc}}$
$\bar{\mu}_{.j} - \bar{\mu}_{.j'}$	$\bar{Y}_{.j.} - \bar{Y}_{.j'.} \pm \boxed{t_{a(b-1)(c-1),\alpha/2}} \sqrt{\mathrm{MS}_{\mathrm{Error}}} \sqrt{\frac{2}{ac}}$
$\mu_{ij} - \mu_{i'j}$	$\bar{Y}_{ij.} - \bar{Y}_{i'j.} \pm t_{\nu^*,\alpha/2} \sqrt{\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}} \sqrt{\frac{2}{c}}$
$\mu_{ij} - \mu_{ij'}$	$\bar{Y}_{ij.} - \bar{Y}_{ij'.} \pm t_{a(b-1)(c-1),\alpha/2} \sqrt{\mathrm{MS}_{\mathrm{Error}}} \sqrt{\frac{2}{c}}$
$\mu_{ij} - \mu_{i'j'}$	$\bar{Y}_{ij.} - \bar{Y}_{i'j'.} \pm t_{\nu^*,\alpha/2} \sqrt{\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}} \sqrt{\frac{2}{c}}$

In the above $\nu^* = \dfrac{\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}}{\frac{\mathrm{MS}^2_{\mathrm{AC}}}{(a-1)(c-1)} + \frac{(b-1)^2\,\mathrm{MS}^2_{\mathrm{Error}}}{a(b-1)(c-1)}}$ à la Satterthwaite[1].

[1] a degrees of freedom approximation when one has not exactly a t-distribution.

Alfalfa data (cont)

Compare Sep 1 date with "no cutting":

$\bar{Y}_{.\text{Sep1}.} =$

$\sqrt{0.02809}$

Dunnett's comparison of marginal date means with *no cutting* as baseline:

$$\bar{Y}_{.j.} - \bar{Y}_{.1.} \pm \boxed{d_{b-1,a(b-1)(c-1),\alpha}} \sqrt{\text{MS}_{\text{Error}}} \sqrt{\frac{2}{ac}}$$

$a = 3$

$c = 6$

Replace, in previous slide, $t_{a(b-1)(c-1),\alpha/2}$ with $d_{b-1,a(b-1)(c-1),\alpha}$.

In Dunnett's table we cannot find $d_{4,45,0.05}$, so take $d_{4,40,0.05} = 2.44$.

$a(b-1)(c-1) = 3 \cdot (4-1)(6-1) = 3 \cdot 3 \cdot 5 = 45$

Table A.5 Critical Values for Dunnett's Two-Sided Test of Treatments versus Control.

Error df	Two-sided α	T = Number of Groups Counting Both Treatments and Control						
		2	3	4	5	6	7	8
5	0.05	2.57	3.03	3.29	3.48	3.62	3.73	3.82
5	0.01	4.03	4.63	4.97	5.22	5.41	5.56	5.68
6	0.05	2.45	2.86	3.10	3.26	3.39	3.49	3.57
6	0.01	3.71	4.21	4.51	4.71	4.87	5.00	5.10
7	0.05	2.36	2.75	2.97	3.12	3.24	3.33	3.41
7	0.01	3.50	3.95	4.21	4.39	4.53	4.64	4.74
8	0.05	2.31	2.67	2.88	3.02	3.13	3.22	3.29
8	0.01	3.36	3.77	4.00	4.17	4.29	4.40	4.48
9	0.05	2.26	2.61	2.81	2.95	3.05	3.14	3.20
9	0.01	3.25	3.63	3.85	4.01	4.12	4.22	4.30
10	0.05	2.23	2.57	2.76	2.89	2.99	3.07	3.14
10	0.01	3.17	3.53	3.74	3.88	3.99	4.08	4.16
11	0.05	2.20	2.53	2.72	2.84	2.94	3.02	3.08
11	0.01	3.11	3.45	3.65	3.79	3.89	3.98	4.05
12	0.05	2.18	2.50	2.68	2.81	2.90	2.98	3.04
12	0.01	3.05	3.39	3.58	3.71	3.81	3.89	3.96
13	0.05	2.16	2.48	2.65	2.78	2.87	2.94	3.00
13	0.01	3.01	3.33	3.52	3.65	3.74	3.82	3.89
14	0.05	2.14	2.46	2.63	2.75	2.84	2.91	2.97
14	0.01	2.98	3.29	3.47	3.59	3.69	3.76	3.83
15	0.05	2.13	2.44	2.61	2.73	2.82	2.89	2.95
15	0.01	2.95	3.25	3.43	3.55	3.64	3.71	3.78
16	0.05	2.12	2.42	2.59	2.71	2.80	2.87	2.92
16	0.01	2.92	3.22	3.39	3.51	3.60	3.67	3.73
17	0.05	2.11	2.41	2.58	2.69	2.78	2.85	2.90
17	0.01	2.90	3.19	3.36	3.47	3.56	3.63	3.69
18	0.05	2.10	2.40	2.56	2.68	2.76	2.83	2.89
18	0.01	2.88	3.17	3.33	3.44	3.53	3.60	3.66
19	0.05	2.09	2.39	2.55	2.66	2.75	2.81	2.87
19	0.01	2.86	3.15	3.31	3.42	3.50	3.57	3.63
20	0.05	2.09	2.38	2.54	2.65	2.73	2.80	2.86
20	0.01	2.85	3.13	3.29	3.40	3.48	3.55	3.60
25	0.05	2.06	2.34	2.50	2.61	2.69	2.75	2.81
25	0.01	2.79	3.06	3.21	3.31	3.39	3.45	3.51
30	0.05	2.04	2.32	2.47	2.58	2.66	2.72	2.77
30	0.01	2.75	3.01	3.15	3.25	3.33	3.39	3.44
40	0.05	2.02	2.29	2.44	2.54	2.62	2.68	2.73
40	0.01	2.70	2.95	3.09	3.19	3.26	3.32	3.37
60	0.05	2.00	2.27	2.41	2.51	2.58	2.64	2.69
60	0.01	2.66	2.90	3.03	3.12	3.19	3.25	3.29

This table produced from the SAS System using function PROBMC('DUNNETT2',,$1-\alpha$,df,k), where $k = T - 1$.

45?

2.44

Figure 1: Table A.5 from Mohr, Wilson, and Freund (2021)

```r
y.1.bar <- mean(alfalfa$yield[alfalfa$date=="none"])
y.2.bar <- mean(alfalfa$yield[alfalfa$date=="sep01"])
y.3.bar <- mean(alfalfa$yield[alfalfa$date=="sep20"])
y.4.bar <- mean(alfalfa$yield[alfalfa$date=="oct07"])


se <- sqrt(MSE) * sqrt(2/(a*c))
me <- 2.44 * se

dtab <- rbind(c(y.2.bar - y.1.bar - me,y.2.bar - y.1.bar + me),
              c(y.3.bar - y.1.bar - me,y.3.bar - y.1.bar + me),
              c(y.4.bar - y.1.bar - me,y.4.bar - y.1.bar + me))

colnames(dtab) <- c("lower","upper")
rownames(dtab) <- c("sep01 - none",
                    "sep20 - none",
                    "oct07 - none")
round(dtab,3)
```

```
              lower  upper
sep01 - none -0.578 -0.305
sep20 - none -0.343 -0.070
oct07 - none -0.226  0.046
```

ARE different

No difference

Unadjusted CIs with `ls_means()` from R package `lmerTest`:

```
ls_means(lmer_out,which="date")
```

```
Least Squares Means table:

          Estimate Std. Error  df t value   lower   upper  Pr(>|t|)
datenone   1.78111    0.11255 6.1  15.825 1.50641 2.05581 3.684e-06 ***
dateoct07  1.69111    0.11255 6.1  15.026 1.41641 1.96581 5.010e-06 ***
datesep01  1.33944    0.11255 6.1  11.901 1.06474 1.61415 1.977e-05 ***
datesep20  1.57444    0.11255 6.1  13.989 1.29974 1.84915 7.646e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

  Confidence level: 95%
  Degrees of freedom method: Satterthwaite
```

```r
ls_means(lmer_out, pairwise = TRUE, which = "date")
```

```
Least Squares Means table:

                        Estimate Std. Error df t value     lower     upper
datenone - dateoct07   0.0900000  0.0558639 45  1.6111 -0.0225156  0.2025156
datenone - datesep01   0.4416667  0.0558639 45  7.9061  0.3291511  0.5541823
datenone - datesep20   0.2066667  0.0558639 45  3.6995  0.0941511  0.3191823
dateoct07 - datesep01  0.3516667  0.0558639 45  6.2951  0.2391511  0.4641823
dateoct07 - datesep20  0.1166667  0.0558639 45  2.0884  0.0041511  0.2291823
datesep01 - datesep20 -0.2350000  0.0558639 45 -4.2067 -0.3475156 -0.1224844
                       Pr(>|t|)
datenone - dateoct07  0.1141598
datenone - datesep01  4.723e-10 ***
datenone - datesep20  0.0005859 ***
dateoct07 - datesep01 1.138e-07 ***
dateoct07 - datesep20 0.0424494 *
datesep01 - datesep20 0.0001219 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

  Confidence level: 95%
  Degrees of freedom method: Satterthwaite
```

References

Mohr, Donna L, William J Wilson, and Rudolf J Freund. 2021. *Statistical Methods*. Academic Press.