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STAT 516 sp 2024 exam 02

75 minutes, no calculators or notes allowed

1. Multiple linear regression

Consider fitting on a data set the multiple linear regression model $Y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip} + \varepsilon_i$, for $i = 1, \ldots, n$, where the ε_i are independent Normal$(0, \sigma^2)$ error terms and the x_{ij} are predictor values.

Suppose the data set has $p = 15$ predictors, but you do not believe all of them are important, so you decide to search for a good model which does not use all 15 predictors.

* (a) Suppose you wish to compare all possible models that one can build from the 15 predictors. How many models will you need to fit?

2^p models

(b) Instead of considering all possible models, you decide to start with the model which uses all the predictors and then to remove one predictor at a time according to some criterion. What is the name for such an approach to model selection?

backward stepwise selection

(c) Give the name of a criterion for comparing models and explain how to use it.

AIC. It is used to estimate how well a model fits the data, while punishing it for having too many parameters. A lower value is interpreted as better

(d) Explain *why* one would wish to discard some of the 15 predictors. Why not just leave all of 15 of them in the model?

Because having too many predictors results in a high chance of multicollinearity and variance inflation factor in the model. The result is that it becomes hard to discern the effects of each feature on the ~~[illegible]~~ final prediction.

2. One-way ANOVA

A study recorded the tensile strengths of sheet metal specimens sampled from four suppliers. A manufacturer wishes to know whether the mean tensile strength differs across these suppliers.

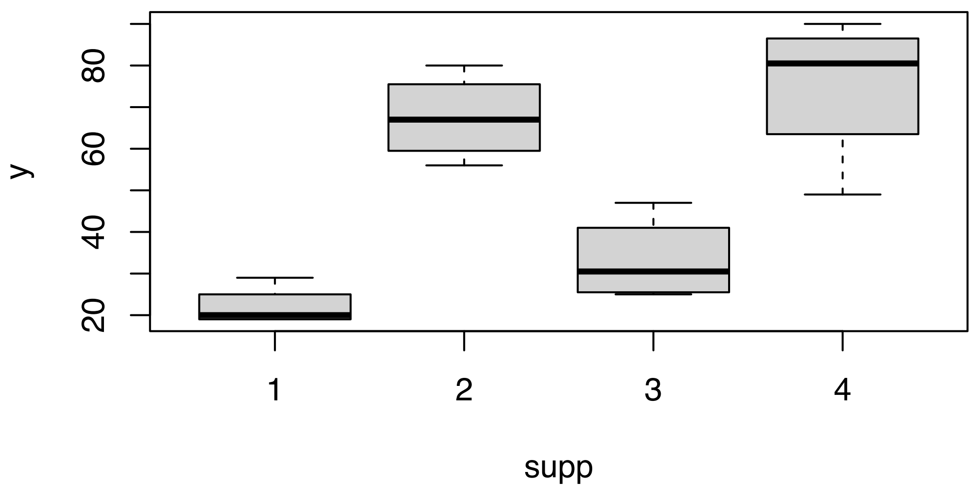
To answer the manufacturer's question, you fit the model

$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij}, \quad i = 1, \dots, a, \quad j = 1, \dots, n,$$

where the ε_{ij} are independent Normal$(0, \sigma^2)$ random variables.

Here is some R output:

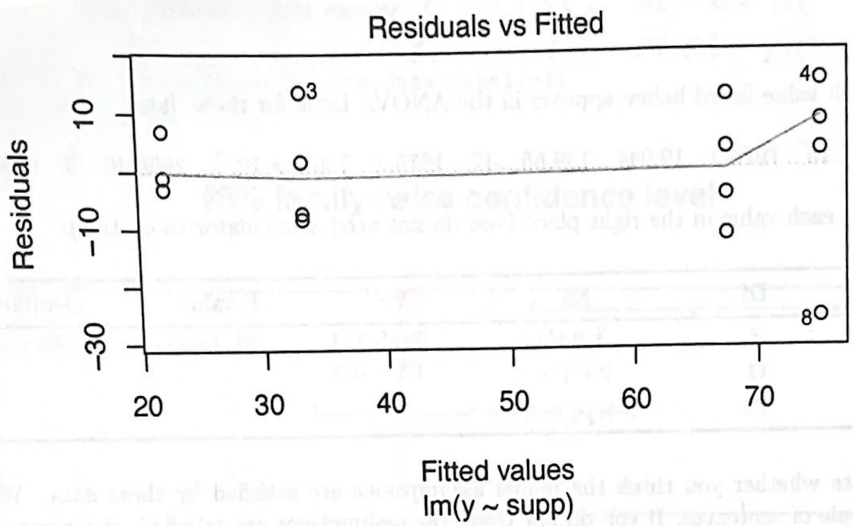
```r
tensile <- data.frame( y = c(19,80,47,90,21,71,26,49,19,63,25,83,29,56,35,78),
                       supp = as.factor(c(1,2,3,4,1,2,3,4,1,2,3,4,1,2,3,4)))
boxplot(y ~ supp, data = tensile)
```



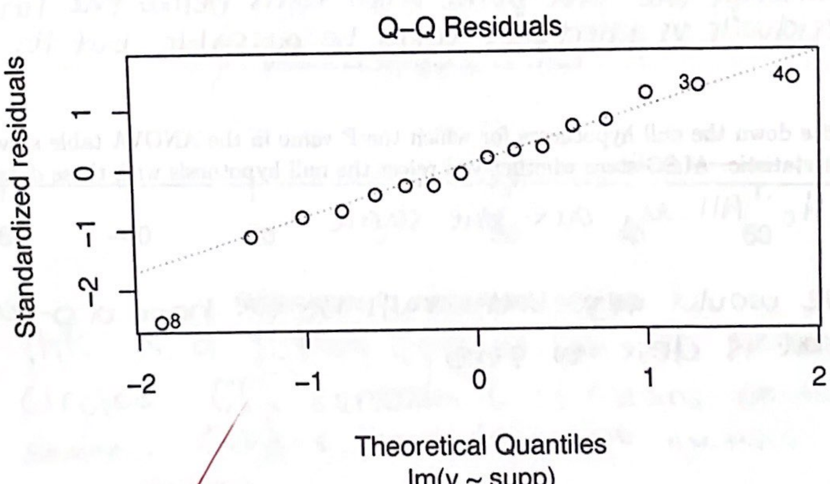

```r
lm_out <- lm(y ~ supp, data = tensile)
lm_out
```

```
Call:
lm(formula = y ~ supp, data = tensile)

Coefficients:
(Intercept)        supp2        supp3        supp4
      22.00        45.50        11.25        53.00
```

```
# summary(lm_out)
# anova(lm_out)
plot(lm_out, which = 1)
```




```
plot(lm_out, which = 2)
```



(a) Give a and n for these data.

a = 4

n = 4

(b) Give each of the treatment group means $\bar{Y}_{i.}$ for $i = 1, 2, 3, 4$ using the estimated model coefficients (you do not need a calculator to do this).

$\bar{y}_{i1} = 22.00$ $(\overset{\hat{\mu}}{22} + 0)$ $\bar{y}_{i4} = 75$ $(22 + \overset{supp4}{53})$

$\bar{y}_{i2} = 67.50$ $(22 + \overset{supp2}{45.5})$

$\bar{y}_{i3} = 33.25$ $(22 + \overset{supp3}{11.25})$

(c) Each value listed below appears in the ANOVA table for these data.

~~15~~ 7978.2 19.044 139.65 ~~12~~ 1675.7 7.401×10^{-5} ~~2659.40~~ ~~3~~ 9653.9

Put each value in the right place (you do not need a calculator to do this):

Source	Df	SS	MS	F value	p-value
Supplier	3	7978.2	2659.4	19.044	7.401×10^{-5}
Error	12	1675.7	139.65		
Total	15	9653.9			

(d) State whether you think the model assumptions are satisfied by these data. Write a couple of sentences. If you do not think the assumptions are satisfied, give some advice about what to do. I think the assumptions look okay. The QQ plot looks good outside of mostly the first point, although the last point also falls below the line. The residuals vs fitted plot could be passable but its not great.

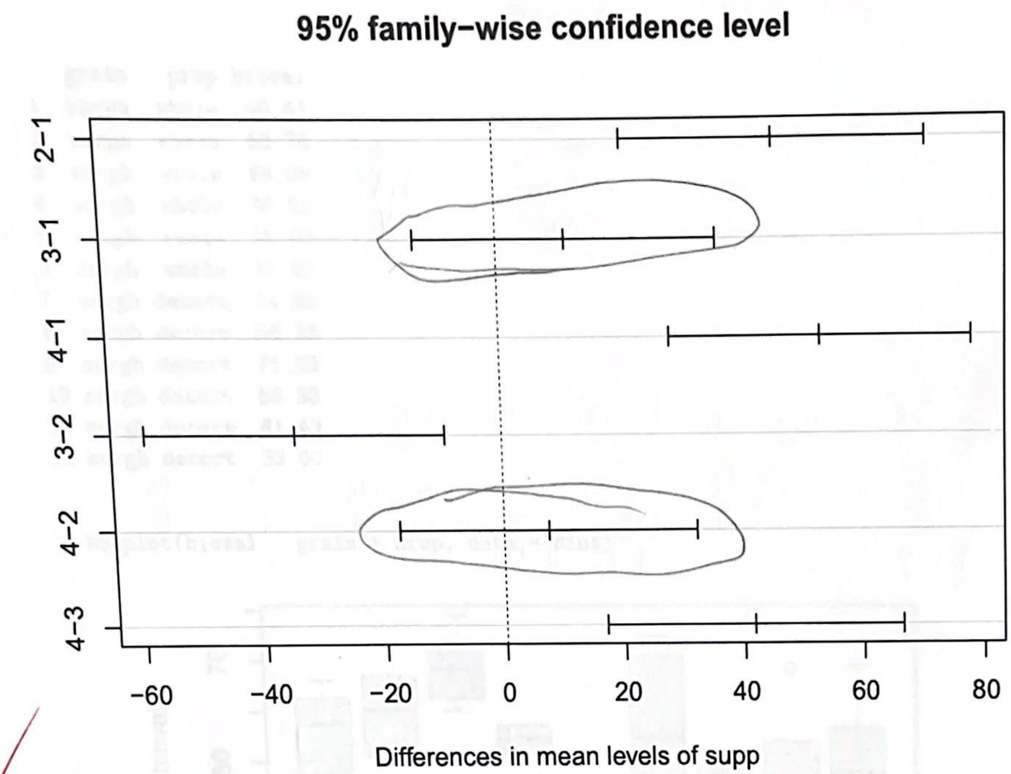
(e) Write down the null hypotheses for which the F value in the ANOVA table serves as a test statistic. ALSO state whether you reject the null hypothesis with these data.

H_0: All μ_i are the same

we would reject the null bc we have a p-value that is close to zero

(f) Assuming the assumptions are satisfied, write (three or four sentences) an interpretation of the output of the plot below. What can you tell the manufacturer about the differences in mean tensile strength between the four manufacturers? Does a ranking of the suppliers emerge? Can you relate this picture to the boxplots shown earlier in this question? Address such questions in your answer.

```
Tukey_out <- TukeyHSD(aov(y~supp,data=tensile))
plot(Tukey_out,cex = .5)
```



Based on the plot, it is determined that I can fail to reject 3-1 and 4-2 and I reject 4-3, 3-2, 4-1, and 2-1. There is significant difference between means 4-3, 3-2, 4-1, and 2-1 and there is not significant difference 4-2 and 3-1. Based on the boxplots, there is more similarity between products (1 and 3) and (4 and 2). There is a ranking of suppliers in the sense that supp 2 and 4 have higher strength than supp 1 and 3.

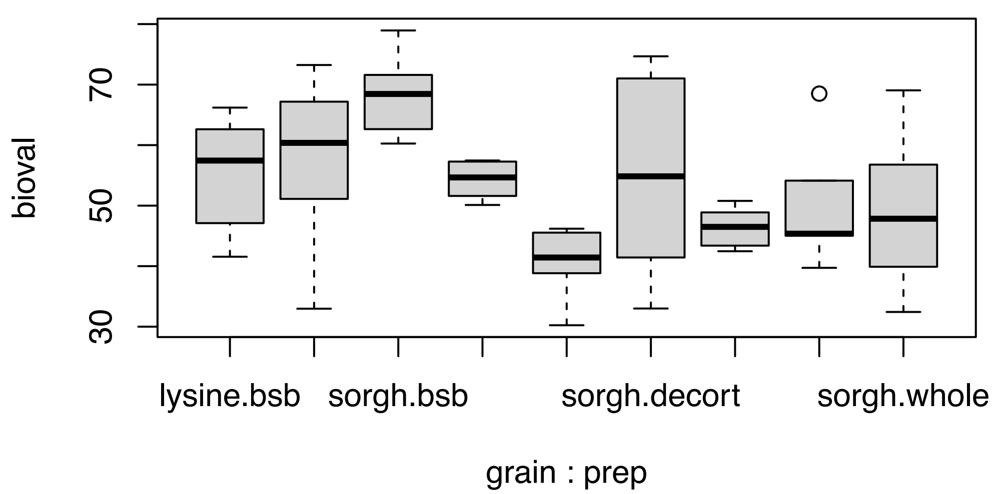
3. Two-way factorial design

Fifty-four rats were randomly assigned to receive one of nine diets such that six rats were assigned to each diet. All combinations of three grain types (sorghum, high-lysine sorghum, millet) and three preparations (whole; decorticated; decorticated, boiled, and soaked) comprised the nine diets. The response for each rat is a biological measurement taken after the rat was fed the diet for some amount of time.

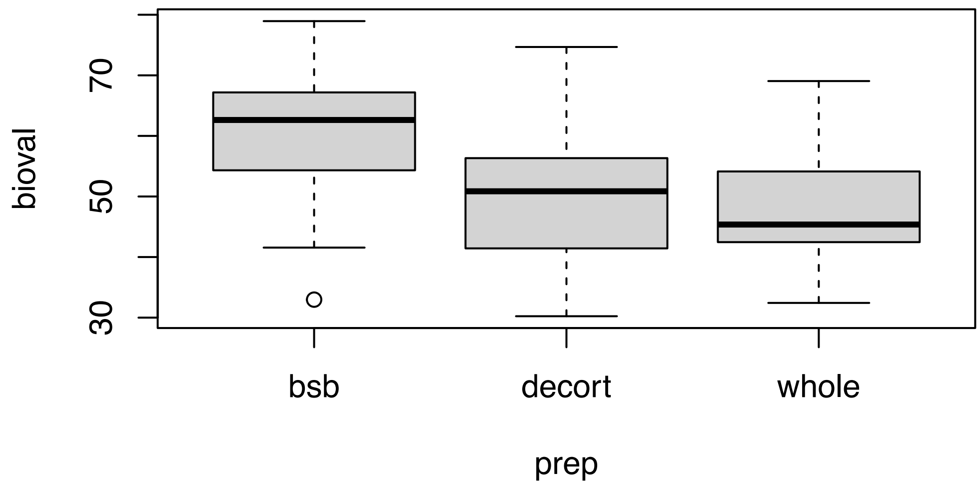
```
head(diet,n=12)
```

```
   grain   prep bioval
1  sorgh  whole  40.61
2  sorgh  whole  56.78
3  sorgh  whole  69.05
4  sorgh  whole  39.90
5  sorgh  whole  55.06
6  sorgh  whole  32.43
7  sorgh decort  74.68
8  sorgh decort  56.33
9  sorgh decort  71.02
10 sorgh decort  53.35
11 sorgh decort  41.43
12 sorgh decort  33.00
```

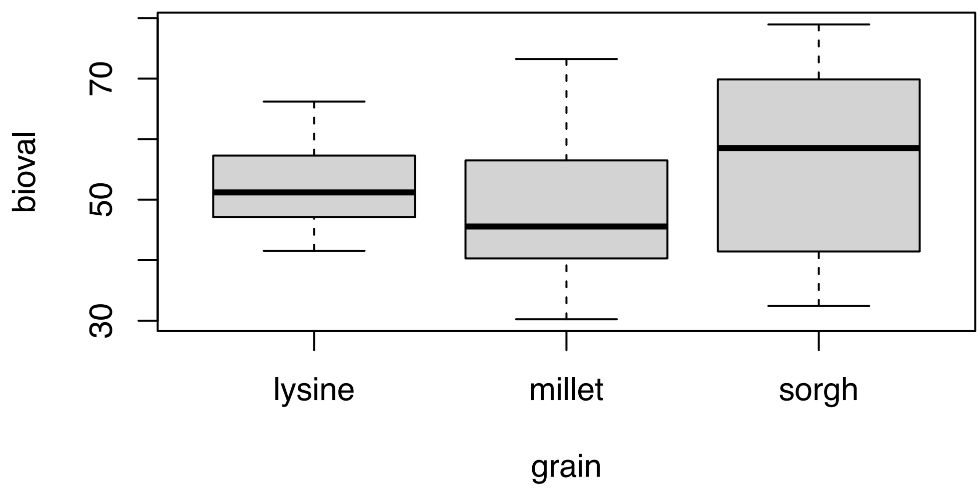
```
boxplot(bioval ~ grain + prep, data = diet)
```



```r
boxplot(bioval ~ prep, data = diet)
```




```r
boxplot(bioval ~ grain, data = diet)
```



Consider modeling the data with the two-way treatment effects model

$$Y_{ijk} + \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij} + \varepsilon_{ijk}, \quad i = 1, \dots, a, \quad j = 1, \dots, b, \quad k = 1, \dots, n_{ij},$$

where the ε_{ijk} are independent Normal$(0, \sigma^2)$ random variables.

(a) Let the grain type be factor A and the preparation type be factor B. Give a and b as well as n_{ij} for all i, j.

$a=3$
$b=3$

$n_{ij} = n = 6$ for all $i=1,\dots,a$
(balanced) $j=1,\dots,n_i$

(b) Use the estimated coefficients (printed below) from the two-way treatment effects model to write an expresseion giving the mean of the responses in the group of rats fed the diet at the factor level combination sorghum × whole (you do not have to evaluate your expression).

$y_{ijk} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij} + \varepsilon_{ijk}$

$\bar{y}_{ij} = \hat{\mu} + 12.997 - 8.982 - 10.440$

$\bar{y}_{ij} = 55.397 + 12.997 - 8.982 - 10.440$

if have to give an estimation for μ

```
lm_out <- lm(bioval ~ grain + prep + grain:prep, data = diet)
lm_out
```

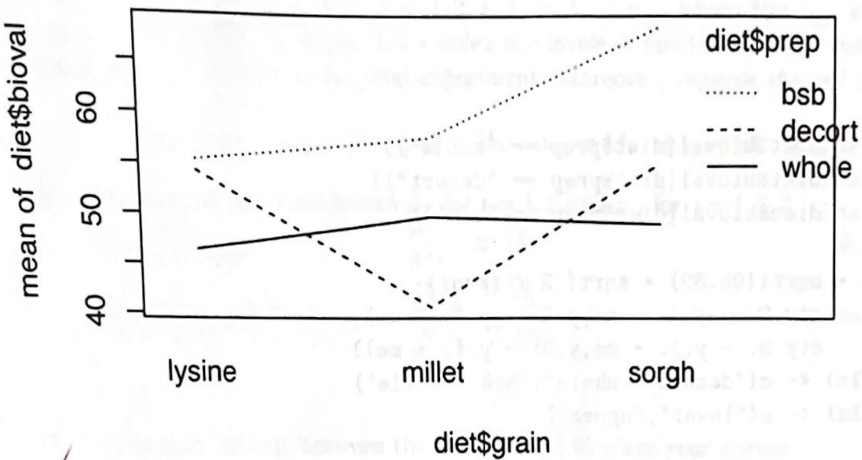
```
Call:
lm(formula = bioval ~ grain + prep + grain:prep, data = diet)

Coefficients:
           (Intercept)                 grainmillet                  grainsorgh
                55.397                       2.145                      12.997
            prepdecort                   prepwhole     grainmillet:prepdecort
                -1.102                      -8.982                     -15.833
grainsorgh:prepdecort     grainmillet:prepwhole       grainsorgh:prepwhole
               -12.323                       1.127                     -10.440
```

(c) Fill in the missing Df values in the ANOVA table below.

	Source	Df	SS	MS	F value	p value
$a-1$	A	2	SS_A	MS_A	2.9334	0.06346
$b-1$	B	2	SS_B	MS_B	7.3265	0.00176
$(a-1)(b-1)$	AB	4	SS_{AB}	MS_{AB}	1.8531	0.13533
$N-ab$	Error	45	SS_{Error}	105.89		
$N-1$	Total	53	SS_{Tot}			

$54-(3\times3)$
$54-9$

(d) In light of the results in the ANOVA table give a careful interpretation of the plot below (more than one sentence).



This interaction plot indicates there may be some interaction. However, from the ANOVA table, we can see that AB has a p-value of 0.13533, meaning there is no significant evidence of interaction. This interaction plot is likely just random noise and there is no evidence of interaction in the main factors.

(e) The p-values 0.06346 and 0.00176 appear in the ANOVA table above. Carefully write down the null hypotheses to which these two p-values correspond.

(Factor A) Ho: Sorghum = High-lysine Sorghum = Millet

HA: At least one of the grains has a different effect

P-value: 0.06346

(Factor B) Ho: Whole = Decorticated = Decorticated, Boiled, Soaked

HA: At least one of the preparation types has a different effect

P-value: 0.00176

(f) What can we conclude on the basis of the p-value which is equal to 0.00176? What does it mean in the terms of the study?

From this p-value, we can conclude that there is significant evidence to reject the null hypothesis of Factor B. In context, this means that there is significant evidence that the preparation type (whole/decorticated/decorticated, boiled, soaked) does have an effect on the outcome of the diet.

(g) Explain in detail what the following code is doing. Give also a careful interpretation of the printed output. Write a few sentences.

```
a <- 3
n <- 6

y.1. <- mean(diet$bioval[diet$prep == "whole"])
y.2. <- mean(diet$bioval[diet$prep == "decort"])
y.3. <- mean(diet$bioval[diet$prep == "bsb"])

me <- 2.29 * sqrt(105.89) * sqrt( 2 / (a*n))
CIs <- rbind(c(y.2. - y.1. - me,y.2. - y.1. + me),
             c(y.3. - y.1. - me,y.3. - y.1. + me))
rownames(CIs) <- c("decort - whole","bsb - whole")
colnames(CIs) <- c("lower","upper")
CIs
```

```
                   lower     upper
decort - whole -6.256030  9.453808
bsb - whole     4.231192 19.941030
```

This code is showing Dunnet's method of comparing means as it compares the means of bsb & decort to a baseline (mean of whole), resulting in a-1 confidence intervals. bsb-whole does not contain 0 so this tells us these means are different, whereas decort-whole does contain 0 meaning these means could be the same.

4. Cell-means model for the two-way factorial design

Let $Y_{ijk} = \mu_{ij} + \varepsilon_{ijk}$, $i = 1, 2$, $j = 1, 2, 3$, $k = 1, \ldots, n_{ij}$, where the ε_{ijk} are independent Normal$(0, \sigma^2)$ random variables. Let i index the levels of one factor and j index the levels of another factor in a two-way factorial experiment. Moreover, suppose the cell means are

$$\mu_{11} = 18, \quad \mu_{12} = 20, \quad \mu_{13} = 21, \quad \mu_{21} = 14, \quad \mu_{22} = 16, \quad \mu_{23} = 17.$$

(a) Compute the marginal means $\bar{\mu}_{i.}$ for $i = 1, 2$ and $\bar{\mu}_{.j}$ for $j = 1, 2, 3$.

$\mu_{1\cdot} = \frac{\mu_{11} + \mu_{12} + \mu_{13}}{3} = \frac{18+20+21}{3} = \frac{59}{3}$ 20

$\mu_{2\cdot} = \frac{\mu_{21} + \mu_{22} + \mu_{23}}{3} = \frac{14+16+17}{3} = \frac{47}{3}$ 16

$\mu_{\cdot 1} = \frac{\mu_{11} + \mu_{21}}{2} = \frac{18+14}{2} = 16$

$\mu_{\cdot 2} = \frac{\mu_{12} + \mu_{22}}{2} = \frac{20+16}{2} = 18$

$\mu_{\cdot 3} = \frac{\mu_{13} + \mu_{23}}{2} = \frac{21+17}{2} = 19$

(b) Is there interaction between the two factors? Explain your answer.

There is not an interaction because the slopes of the plot are identical. There are likely significant main effects of both, but the interaction does not affect the response.

(c) Carefully draw an interaction plot with the level $j = 1, 2, 3$ along the horizontal axis.