STAT 516 sp 2025 exam 02

75 minutes, no calculators, two pages of notes (one-sided)

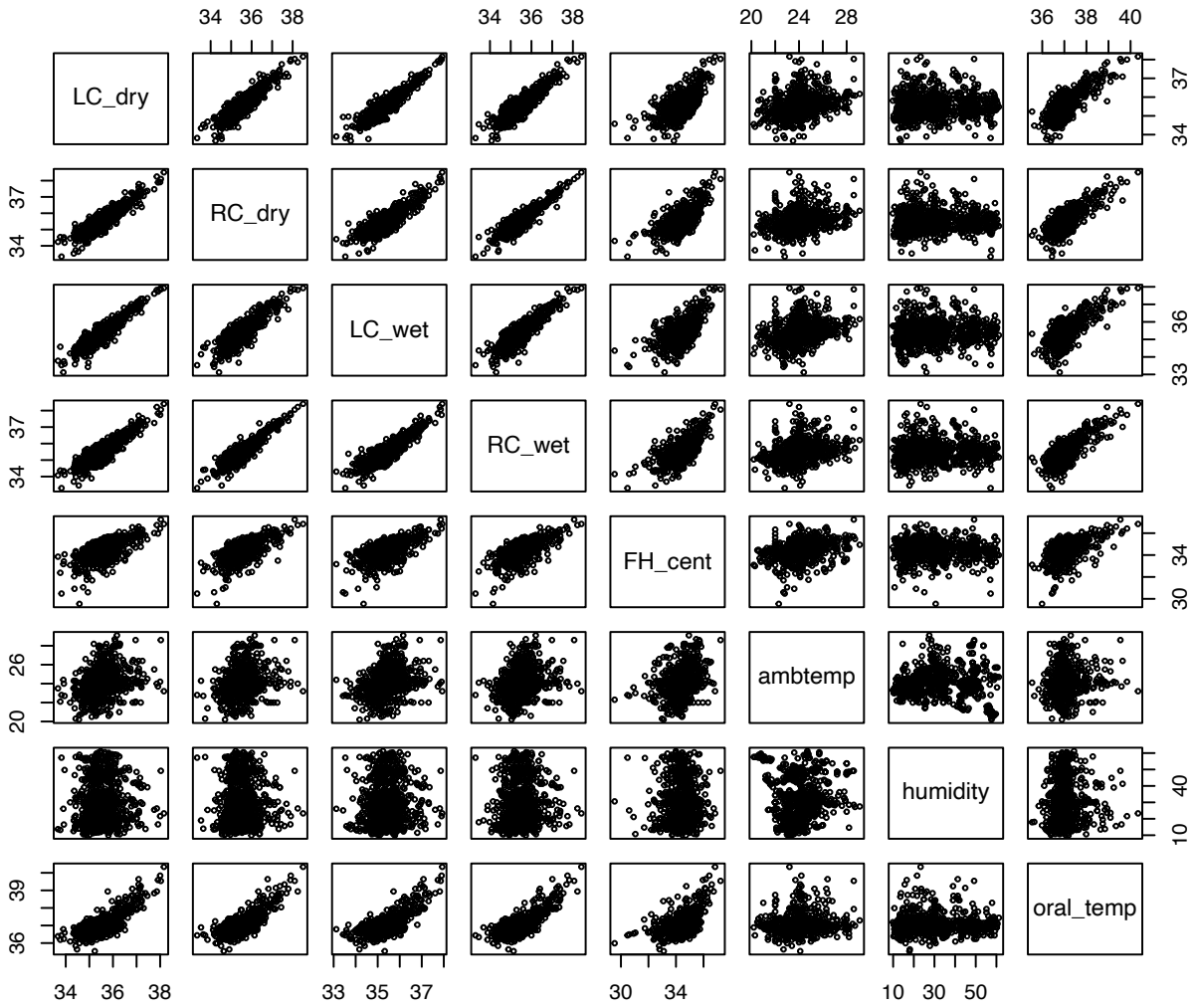
1. Multiple linear regression

In a study of the accuracy of infrared thermography (IRT) to determine humans' body temperatures from a thermal image of the face, the oral temperatures (regarded as the correct temperatures) of 933 subjects were recorded as well as the temperature readings from IRT at various regions of the subjects faces. Also recorded were the humidity level and the ambient temperature of the environment in which the IRT measurements were taken as well as the distance of the subject from the infrared camera. The table below describes the variables in the data set:

Variable	Description
`LC_Dry`	IRT temperature at dry area of left canthus
`LC_Wet`	IRT temperature at wet area of left canthus
`RC_Dry`	IRT temperature at dry area of right canthus
`RC_Wet`	IRT temperature at wet area of right canthus
`FH_cent`	IRT temperature at center of forehead
`ambtemp`	The ambient temperature
`humidity`	The humidity level
`distance`	Distance of the subject to the thermal camera
`oral_temp`	The subject's temperature as measured with an oral thermometer (the response)

Study carefully the R code and its output below:

```
plot(data, cex=.5)
```




```
lm1 <- lm(oral_temp ~ LC_wet + FH_cent + ambtemp + humidity, data = data)
summary(lm1)
```

```
Call:
lm(formula = oral_temp ~ LC_wet + FH_cent + ambtemp + humidity,
    data = data)

Residuals:
    Min      1Q  Median      3Q     Max
-1.3257 -0.2249 -0.0386  0.1951  1.6777

Coefficients:
```

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 16.1919941  0.6874924  23.552  < 2e-16 ***
LC_wet       0.5471070  0.0242316  22.578  < 2e-16 ***
FH_cent      0.0838784  0.0190259   4.409 1.16e-05 ***
ambtemp     -0.0597091  0.0093124  -6.412 2.29e-10 ***
humidity     0.0006504  0.0009033   0.720    0.472
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3593 on 928 degrees of freedom
Multiple R-squared:  0.5074,    Adjusted R-squared:  0.5053
F-statistic:   239 on 4 and 928 DF,  p-value: < 2.2e-16
```

```r
lm2 <- lm(oral_temp ~ LC_wet + LC_dry + RC_wet + RC_dry
          + FH_cent + ambtemp + humidity, data = data)
summary(lm2)
```

```
Call:
lm(formula = oral_temp ~ LC_wet + LC_dry + RC_wet + RC_dry +
    FH_cent + ambtemp + humidity, data = data)

Residuals:
     Min       1Q   Median       3Q      Max
-1.13412 -0.20883 -0.02978  0.19202  1.68678

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 13.1700628  0.6634732  19.850  < 2e-16 ***
LC_wet       0.0199131  0.0479429   0.415    0.678
LC_dry       0.2459987  0.0577504   4.260 2.26e-05 ***
RC_wet       0.2326471  0.0505648   4.601 4.79e-06 ***
RC_dry       0.2035901  0.0494070   4.121 4.12e-05 ***
FH_cent      0.0056129  0.0181624   0.309    0.757
ambtemp     -0.0537898  0.0084738  -6.348 3.42e-10 ***
humidity     0.0009555  0.0008222   1.162    0.245
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3261 on 925 degrees of freedom
Multiple R-squared:  0.5956,    Adjusted R-squared:  0.5926
F-statistic: 194.6 on 7 and 925 DF,  p-value: < 2.2e-16
```

```
library(car)
```

```
Warning: package 'car' was built under R version 4.4.1
```

```
Loading required package: carData
```

```
vif(lm1)
```

```
  LC_wet  FH_cent  ambtemp humidity
1.626828 1.619943 1.154916 1.017395
```

```
vif(lm2)
```

```
  LC_wet   LC_dry   RC_wet   RC_dry  FH_cent  ambtemp humidity
7.733028 9.891932 8.289578 7.914788 1.792566 1.161182 1.023502
```

Note that two models were fit: In the first model, only one of the four variables `LC_Dry`, `LC_Wet`, `RC_Dry`, and `RC_Wet` were included, whereas in the second model, all four of these variables were included as predictors.

(a) Report the value of R^2 for both models, and explain why it is higher for one model than for the other.

In the first model $R^2 = 0.5074$ In the second model $R^2 = 0.5956$

The R^2 is higher in the second model because there are more variables.

(b) Report the p-value for testing the significance of `LC_Wet` in both models. Does one come to the same conclusion regarding the importance of this variable for predicting a subject's oral temperature?

First model p-value = $2e^{-16}$ Second model p-value = 0.678

No, we get different answers in each model. In the first model we reject H_0 and in the second model we fail to reject H_0. When LC_wet was the only variable it was statistically significant, but once the other three variables were added it was no longer statistically significant.

(c) Study carefully the figure displaying scatterplots for every pair of variables in the data set. How can this scatterplot help you understand your observation from part (b)? Give a detailed answer.

The scatterplot shows how linearly related the covariates are to each other. When two or more covariates have a strong linear relationship and they are both in the model, it can make the covariates seem less significant than they actually are.

(d) Name two strategies we talked about in class for selecting a set of variables to keep in the model.

Foward stepwise selection

Backward stepwise selection

(e) Give one reason why one might not want to include all available variables in one's model.

If not all the variables are significant to the model or if multiple covariates are linearly related to each other, it may be a good idea to not include all variables and preform variable selection.

(f) Explain the output of `vif(lm1)` and `vif(lm2)`. What is a "VIF" and why did the VIF change for the variable `LC_wet` from the first to the second model?

VIF is the variable inflation factor $\left(\frac{1}{1-R^2}\right)$

The R^2 in the VIF is the one you get when you regress x_j on another covariate. The stronger the linear relationship is between the two, the closer to 1 R^2 gets, and the bigger the VIF becomes. This results in wider CI's and makes p values of these covariates go up.

The VIF of LC_wet increased from lm1 to lm2 because lm2 included multiple covariates that LC_wet has a strong linear relationship to that were not included in lm1.

2. One-way ANOVA

An experiment studied the effect of temperature on the failure time of a kind of sheathed tubular heater. At each of four temperatures, 1520°, 1620°, 1660°, and 1708°, the number of hours until failure was recorded for six heaters. The data are tabulated here:

Temperature	Failure time (hrs)
1520°	1953,2135,2471,4727,6134,6314
1620°	1190,1286,1550,2125,2557,2845
1660°	651,837,848,1038,1361,1543
1708°	511,651,651,652,688,729

Consider fitting the one-way ANOVA model to these data. Let

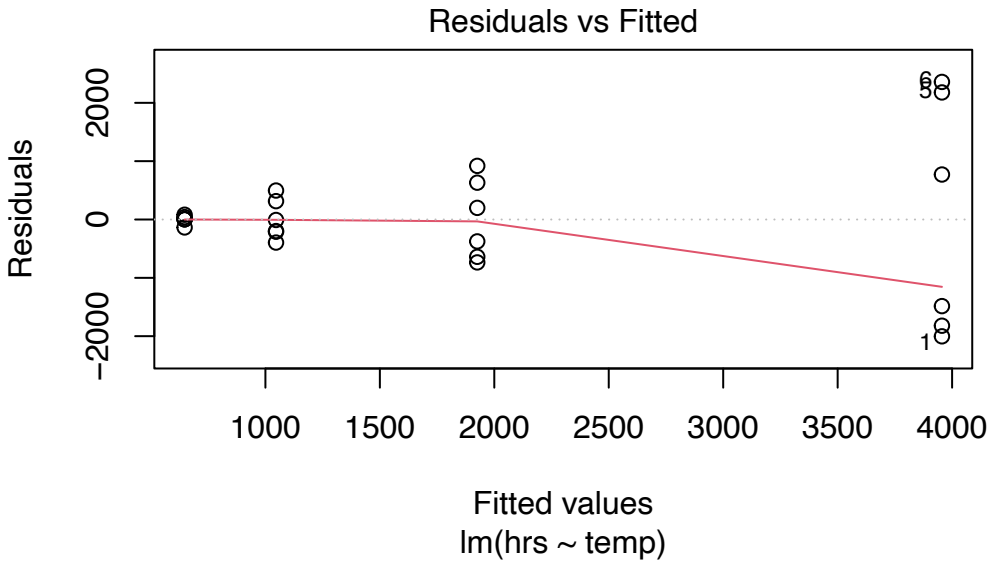
$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij}$$

for $i = 1, \ldots, a$, $j = 1, \ldots, n_i$, where the ε_{ij} are independent Normal$(0, \sigma^2)$ random variables.

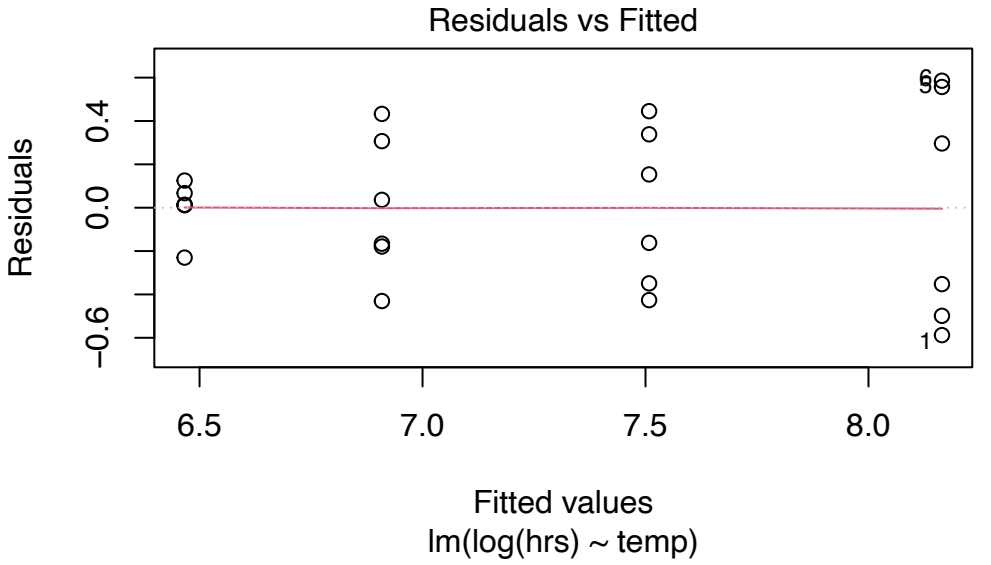
The R code below reads in the data and fits two one-way ANOVA models: One using the original response values and one using the natural log of the response values. Residuals versus fitted values plots for the two models are shown.

```
hrs <- c(1953,2135,2471,4727,6134,6314,
         1190,1286,1550,2125,2557,2845,
         651,837,848,1038,1361,1543,
         511,651,651,652,688,729)
temp <- as.factor(c(rep(1520,6),rep(1620,6),rep(1660,6),rep(1708,6)))

lm_hrs <- lm(hrs~temp)
plot(lm_hrs,which = 1)
```




```
lm_loghrs <- lm(log(hrs)~temp)
plot(lm_loghrs,which = 1)
```




```
summary(lm_loghrs)
```

```
Call:
lm(formula = log(hrs) ~ temp)
```

log

```
Residuals:
     Min       1Q   Median       3Q      Max
-0.58769 -0.25978  0.01279  0.29893  0.58571

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   8.1648     0.1507  54.169  < 2e-16 ***
temp1620     -0.6567     0.2132  -3.081  0.00589 **
temp1660     -1.2559     0.2132  -5.892 9.20e-06 ***
temp1708     -1.6983     0.2132  -7.967 1.24e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3692 on 20 degrees of freedom
Multiple R-squared:  0.7823,    Adjusted R-squared:  0.7497
F-statistic: 23.96 on 3 and 20 DF,  p-value: 7.912e-07
```

N=24

$\sqrt{\sigma^2} = \sigma$

$\frac{SS_{Error}}{20}$

$(.3693)^2 = SS_{Total}$

```
TukeyHSD(aov(log(hrs) ~ temp),conf.level = .99)

  Tukey multiple comparisons of means
    99% family-wise confidence level

Fit: aov(formula = log(hrs) ~ temp)

$temp
                 diff       lwr         upr     p adj
1620-1520 -0.6567415 -1.413104  0.09962083 0.0277405
1660-1520 -1.2558618 -2.012224 -0.49949948 0.0000508
1708-1520 -1.6983332 -2.454696 -0.94197085 0.0000007
1660-1620 -0.5991203 -1.355483  0.15724202 0.0488199
1708-1620 -1.0415917 -1.797954 -0.28522935 0.0004796
1708-1660 -0.4424714 -1.198834  0.31389095 0.1950191
```

1520 > 1660, 1708

1520 = 1620

(a) Explain carefully why the model which uses the natural log of the responses will probably yield more reliable inferences.

Looking at the residuals vs. fitted graph the ~~original~~ original data has unequal variance that gradually increases with a large difference between the first & last group. The log() graph still has the problem of an increasing variance, but the difference between groups is much smaller. ~~both will make~~

(b) Use the R output to compute the mean of the natural log of the observed failure times in the 1620° temperature group.

7.5081

(c)

$$8.1648 - 0.6567 = 7.5081$$

Source	Df	SS	MS	F	p-value
Treatment	viii	xiv	v	xi	xii
Error	i	x	iii		
Total	ix	xiii			

$MS_{Error} = 0.3692^2 = \frac{SSTr}{20}$

$23.96 = \frac{MSTrT}{0.3692^2}$

Fill the blank ANOVA table with numerals from among (i)–(xx) to indicate which of the below values belong where (more values are listed than are needed):

~~(i) 20~~
(ii) 0.3692
~~(iii) $(0.3692)^2$~~
(iv) 1.24×10^{-7}
(v) 0.7497
~~(vi)~~ $(23.96)(0.3692)^2$
(vii) 8.1648
~~(viii) 3~~
~~(ix) 23~~
~~(x) 20~~$(0.3692)^2$
~~(xi) 23.96~~
~~(xii) 7~~.912 $\times 10^{-7}$
~~(xiii) 20~~$(0.3692)^2 + 3(23.96)(0.3692)^2$
~~(xiv)~~ $3(23.96)(0.3692)^2$
(xv) 0.7823
(xvi) $(23.96)^2$
(xvii) $20(23.96)(0.3692)^2$
(xviii) $20(0.3692)$
(xix) 24
(xx) 17

(d) Based on the model with the natural log of the responses, is there evidence to conclude that the temperature is related to the failure time? Explain your answer carefully.

Yes. The p-val is very low, meaning you reject the null that $T_i = 0$ & say at least one of the group means differ mani; a significant treatment effect.

(e) In the experiment, under which temperature did the sheathed tubular heaters last the longest, on average, before failing? Based on the R output, can we conclude that under this temperature, the mean failure time was statistically significantly greater than the other means? Explain your answer.

1520°. You can conclude it is significantly greater than 1708° & 1660°, but not 1620°, bc in the Tukey table, 1620-1520 conf int contains ∅.

(f) If one wished only to compare the mean failure times at the temperatures 1520° and 1620°, one would construct the confidence interval $\bar{Y}_{1\cdot} - \bar{Y}_{2\cdot} \pm 0.4446392$, where the margin of error involves a quantile from a t-distribution. With Tukey's method, however, the confidence interval for comparing these means is constructed as $\bar{Y}_{1\cdot} - \bar{Y}_{2\cdot} \pm 0.5966148$. Explain the difference between the two intervals and explain the reason for the difference.

The Tukey interval would be wider, as the margin of error must be larger due to the requirement that it must control for Type I error more as it is making more comparisons and familywise coverage is desired.

(g) What additional plot should one generate in order to ensure that the data from this experiment satisfies the assumptions of the one-way ANOVA model?

The normal Q-Q plot to test for responses being normally distributed around treatment means.

3. Two-way factorial design

In order to understand how the temperature and salinity of water effect the growth of shrimp raised in aquariums, three aquiriums were set to each combination of temperatures (25° and 35° Celcius) and salinity levels (10%, 25%, and 40%) and the weight gain of the shrimp over a period of four weeks recorded for each aquarium. The experiment resulted in the data tabulated below:

Temperature	Salinity	Weight gain	$\bar{Y}_{ij\cdot}$
25°	10%	86,52,73	70.33
	25%	544,371,482	465.67
	40%	390,290,397	359.00
35°	10%	439,436,349	408.00
	25%	249,245,330	274.67
	40%	247,277,205	243.00

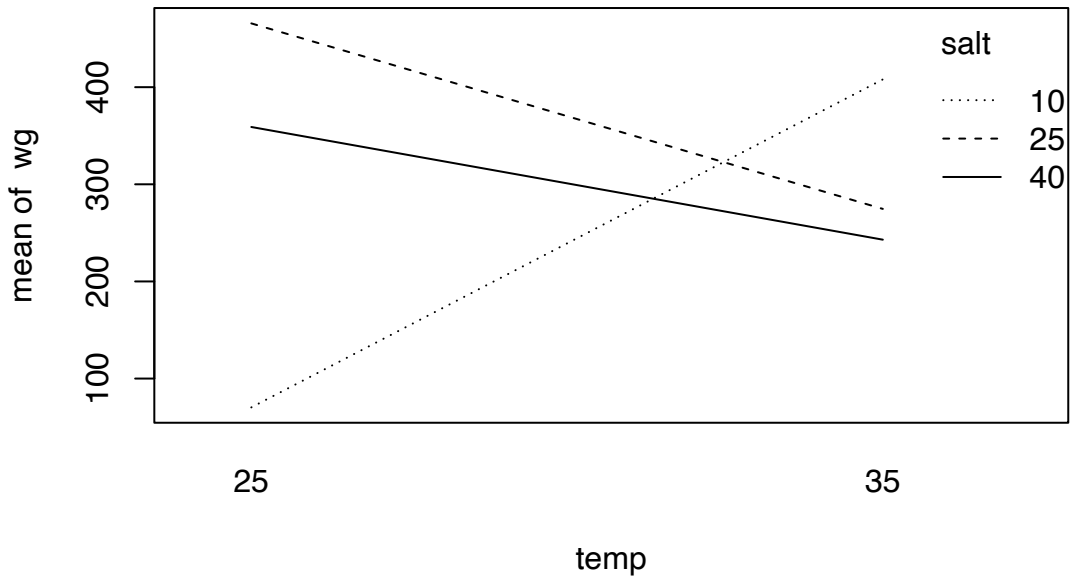
Consider the following model, assuming that the assumptions are satisfied: Let

$$Y_{ijk} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij} + \varepsilon_{ijk},$$

$i = 1, \ldots, a$, $j = 1, \ldots, b$, and $k = 1, \ldots, n_{ij}$, where the ε_{ijk} are independent Normal$(0, \sigma^2)$ random variables. Let i index the temperature and j index the salinity level. Consider the R code below and its output:

```r
wg <- c(86,52,73,544,371,482,390,290,397,
        439,436,349,249,245,330,247,277,205)
temp <- as.factor(c(rep(25,9),rep(35,9)))
salt <- as.factor(c(1,1) %x% c(rep(10,3),rep(25,3),rep(40,3)))

interaction.plot(temp,salt,wg)
```




```r
lm_shrimp <- lm(wg ~ temp + salt + temp:salt)

TukeyHSD_out <- TukeyHSD(aov(lm_shrimp))
TukeyHSD_out$`temp:salt`
```

```
                  diff        lwr       upr        p adj
35:10-25:10  337.66667  188.36777 486.96557 7.262611e-05
25:25-25:10  395.33333  246.03443 544.63223 1.455431e-05
35:25-25:10  204.33333   55.03443 353.63223 6.247420e-03
25:40-25:10  288.66667  139.36777 437.96557 3.297825e-04
35:40-25:10  172.66667   23.36777 321.96557 2.060247e-02
25:25-35:10   57.66667  -91.63223 206.96557 7.812446e-01
35:25-35:10 -133.33333 -282.63223  15.96557 9.059335e-02
25:40-35:10  -49.00000 -198.29890 100.29890 8.713239e-01
35:40-35:10 -165.00000 -314.29890 -15.70110 2.757719e-02
35:25-25:25 -191.00000 -340.29890 -41.70110 1.028917e-02
25:40-25:25 -106.66667 -255.96557  42.63223 2.300708e-01
35:40-25:25 -222.66667 -371.96557 -73.36777 3.185672e-03
25:40-35:25   84.33333  -64.96557 233.63223 4.476993e-01
35:40-35:25  -31.66667 -180.96557 117.63223 9.766950e-01
35:40-25:40 -116.00000 -265.29890  33.29890 1.680994e-01
```

```
anova(lm_shrimp)

Analysis of Variance Table

Response: wg
           Df Sum Sq Mean Sq F value    Pr(>F)
temp        1    470     470  0.1587  0.697379
salt        2  51537   25768  8.6953  0.004633 **
temp:salt   2 245463  122732 41.4144 4.106e-06 ***
Residuals  12  35562    2964
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

a-1 b-1 (a-1)(b-1) ab(n-1)

(a) Fill in the missing values in the above ANOVA table (five values have been removed).

(b) Give the value of $\hat{\sigma}^2$.

$\hat{\sigma}^2 = MS_{error} = 2964$

(c) Give the value from the ANOVA table which reflects the ratio of the variation in the responses owing to the effect of the different temperatures over the variation owing to random differences from aquarium to aquarium.

$F_A = MS_A / MS_{error} = 0.1587$

(d) Give the value of $2\sum_{j=1}^{3} 3(\bar{Y}_{.j.} - \bar{Y}_{...})^2$, which appears in the ANOVA table.

$SS_B = an \sum_{j=1}^{b} (\bar{y}_{\cdot j \cdot} - \bar{y}_{...})^2$, $a=2$, $n=3$ so $SS_B = 51537$

(e) Can one say that one temperature is better than the other? Explain your answer. What would you say if a shrimp supplier asked, "At which temperature should I keep my aquariums?"

No, not as a blanket statement, you can not say 'this temp. is better in all cases'. There is an interaction between temp. and salinity, so it depends on the salinity as well.

(f) If someone said that the temperature is irrelevant to the growth rate of shrimp because of the p-value 0.697379 appearing in the table, what would you say in response?

This p val relates to the main effect of temperature, but we can see that an interaction effect still exists. So, no, temperature is not entirely irrelevant.

(g) Give an interpretation to the value 4.106×10^{-6} appearing in the ANOVA table.

4.0166e-06 is the p-value for interaction effects. If factors A & B did NOT interact, the likelyhood of getting these results in our data would be 4.0166e-06 — very slim.

(h) Based on the R output, can you recommend a single best combination of temperature and salinity for fostering the growth of shrimp? If so, what is it; if not, why not?

Based on the R-output and Tukey's, I cannot recommend a single best combination. The 3 top combos are 25:25, 35:10, and 25:40. Based on Tukey's, none of these 3 combos show significant difference from eachother, and all CI's contain 0.

(i) Based on the R output, can you identify a single worst combination of temperature and salinity for fostering the growth of shrimp? If so, what is it; if not, why not?

25:10 is the single worst combo. The closest mean of wg to 25:10 is 35:40. Based on Tukey's for 35:40 - 25:10, the CI doesn't contain 0, and p-val is ~.02, so these results are significant (with $\alpha = 0$ showing 25:10 as the worst combo.

(j) Suppose one of the aquariums had started leaking during the experiment so that the weight gain of the shrimp in this aquarium had to be excluded from the analysis, resulting in only two values for one of the temperature and salinity combinations. What do we call the situation in which the number of replicates is not the same for all combinations of factor levels? How does this complicate the analysis?

Unbalanced design. Unbalanced design makes it more difficult to obtain the sum of squares for main effects in the analysis. This also complicates the analysis because the anova function in R is programmed to find SSQ sequentially, which would not show a correct output for an unbalanced design. Unbalanced makes analysis more tedious & difficult.