STAT 516 sp 2026 exam 01

75 minutes, one page of handwritten notes allowed, no calculators

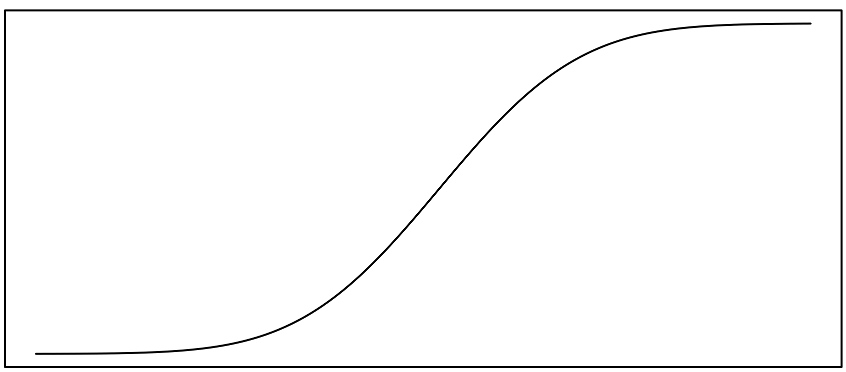
1. Inference on the mean of a normal population

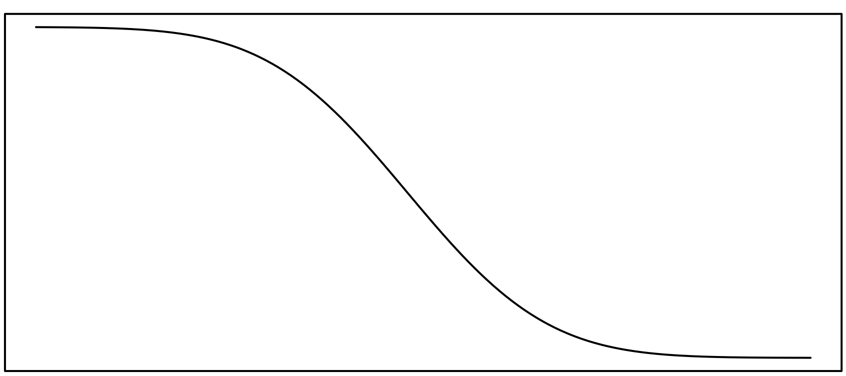
Suppose $X_1, \dots, X_n$ is a random sample from the $\mathcal{N}(\mu, \sigma^2)$ distribution, where σ^2 is known, and that we plan to test H_0: $\mu \geq 5$ versus H_1: $\mu < 5$ with a test that rejects H_0 when

$$Z_{\text{test}} = \frac{\bar{X} - 5}{\sigma/\sqrt{n}} < -z_{0.10},$$

where $z_{0.10}$ is the value such that $P(Z > z_{0.10}) = 0.10$, $Z \sim \mathcal{N}(0, 1)$. Suppose this test would have power 0.80 if μ were equal to 3.

(a) Indicate the panel in which the curve could be the power curve for this test.





(b) Put a tick mark at $y = 0.10$ on the vertical axis and at $x = 5$ on the horizontal axis.

(c) Draw a horizontal line from the $y = 0.10$ tick mark and a vertical line from the $x = 5$ tick mark.

(d) What is the maximum height of the power curve?

(e) What is the minimum height of the power curve?

(f) Draw a horizontal line and having height $y = 0.80$ and, where this crosses the power curve, drop a vertical line down to the horizontal axis; label the position where this vertical line touches the horizontal axis.

(g) What is the probability that this test would make a Type II error if μ were equal to 3?

(h) Suppose the sample size were increased considerably. Draw the new power curve and label it "(h)".

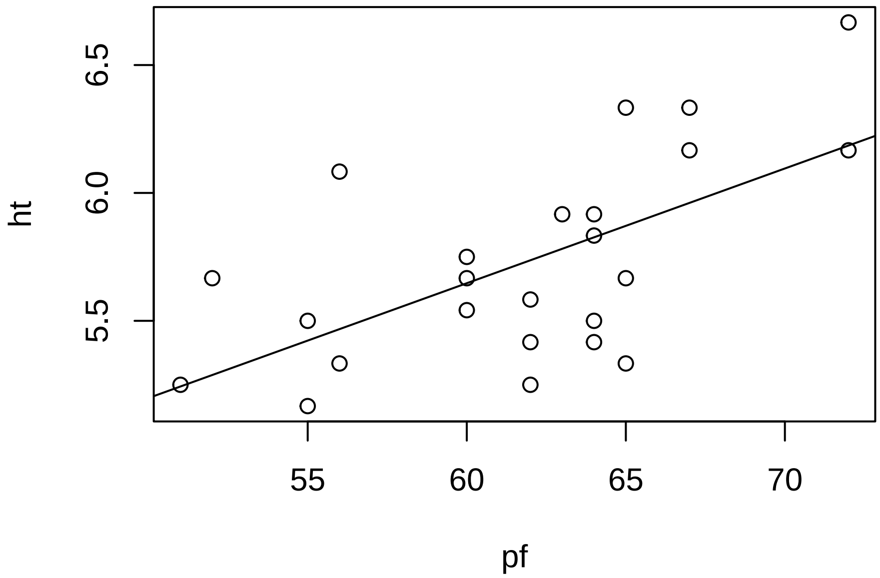
(i) Starting from the original power curve, suppose σ were increased considerably. Draw the new power curve and label it "(i)".

(j) Starting from the original power curve, suppose the significance level of the test were decreased to 0.01. Draw the new power curve and label it "(j)". *You can squeeze this onto the same plot, or you can use the space below to draw a new plot.*

2. Simple linear regression

The scatterplot shows the heights (in feet) and pinky finger lengths (in millimeters) of several students with least-squares line overlaid.

```
plot(ht ~ pf)
abline(lm(ht ~ pf))
```



(a) Circle a point having a residual that is less than zero.

(b) Draw a box around the point appearing to have the residual that is largest in magnitude.

(c) Do your best to locate the position of $(\bar{x}_n, \bar{Y}_n)$ on the plot; draw a dark circle here.

(d) Indicate the point (or points) on the plot with the smallest leverage.

(e) Recall that the least-squares criterion is defined as $Q(b_0, b_1) = \sum_{i=1}^{n}(Y_i - (b_0 + b_1 x_i))^2$ for any candidate intercept b_0 and slope b_1. What name do we give the quantity $Q(\bar{Y}_n, 0)$?

(f) Letting $\hat{\varepsilon}_1, \dots, \hat{\varepsilon}_n$ denote the residuals, what name do we give the quantity $\sum_{i=1}^{n} \hat{\varepsilon}_i^2$?

(g) Give an expression in terms of quantities appearing in the output below for the coefficient of determination R^2 (you do not need to evaluate your expression).

```
anova(lm(ht ~ pf))
```

```
Analysis of Variance Table

Response: ht
          Df Sum Sq Mean Sq F value   Pr(>F)
pf         1 1.4005 1.40054   13.66 0.001262 **
Residuals 22 2.2556 0.10253
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(h) What is the sample size n for this data set?

(i) Give the value of $\hat{\sigma}^2$.

(j) Suppose you find hand prints left by two individuals—one with pinky finger length 65mm and one with pinky finger length 70mm. Of which individual can you more accurately predict the height based on the model fitted to this data set? Carefully explain your answer.

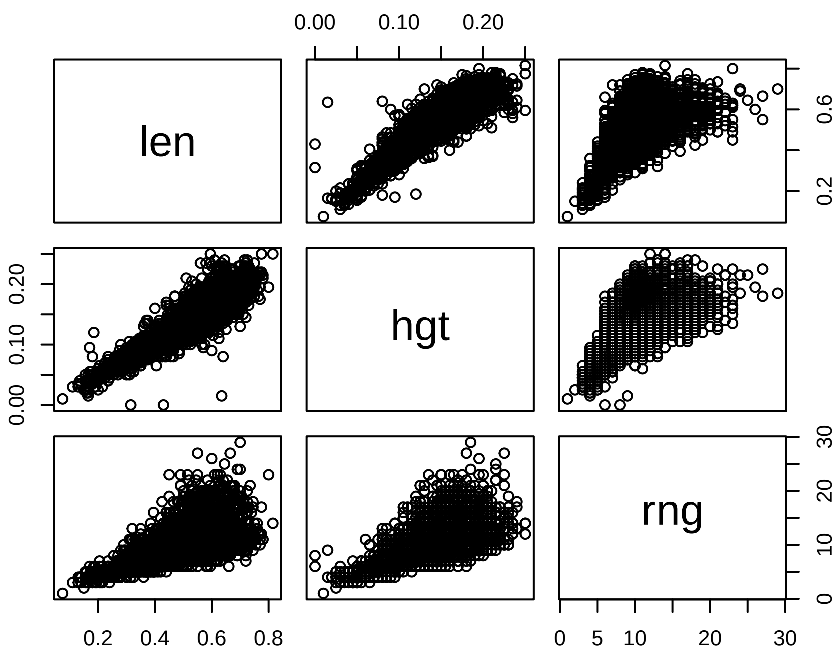
3. Multiple linear regression

A data set of which the first few records are shown below contains length and height measurements as well as the number of rings of a sample of abalones.

```
    len   hgt rng
1 0.350 0.090   7
2 0.530 0.135   9
3 0.440 0.125  10
4 0.330 0.080   7
5 0.425 0.095   8
6 0.530 0.150  20
```

Consider using the multiple linear regression model to predict the number of rings an abalone has (these are like tree rings, and tell the age of the abalone, but counting them requires cutting the abalone open).

```r
plot(ab)
```




```r
lm1 <- lm(rng ~ len + hgt, data = ab)
summary(lm1)
```

```
Call:
lm(formula = rng ~ len + hgt, data = ab)

Residuals:
    Min      1Q  Median      3Q     Max
-6.0433 -1.6385 -0.5679  0.8442 16.6717

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   2.6651     0.1792  14.874   <2e-16 ***
len           1.1304     0.7547   1.498    0.134
hgt          47.9564     2.3550  20.364   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.554 on 4171 degrees of freedom
Multiple R-squared:  0.3729,    Adjusted R-squared:  0.3726
F-statistic:  1240 on 2 and 4171 DF,  p-value: < 2.2e-16
```

(a) Give the value of $\hat{\sigma}$.

(b) Explain what the parameter σ describes in the multiple linear regression model.

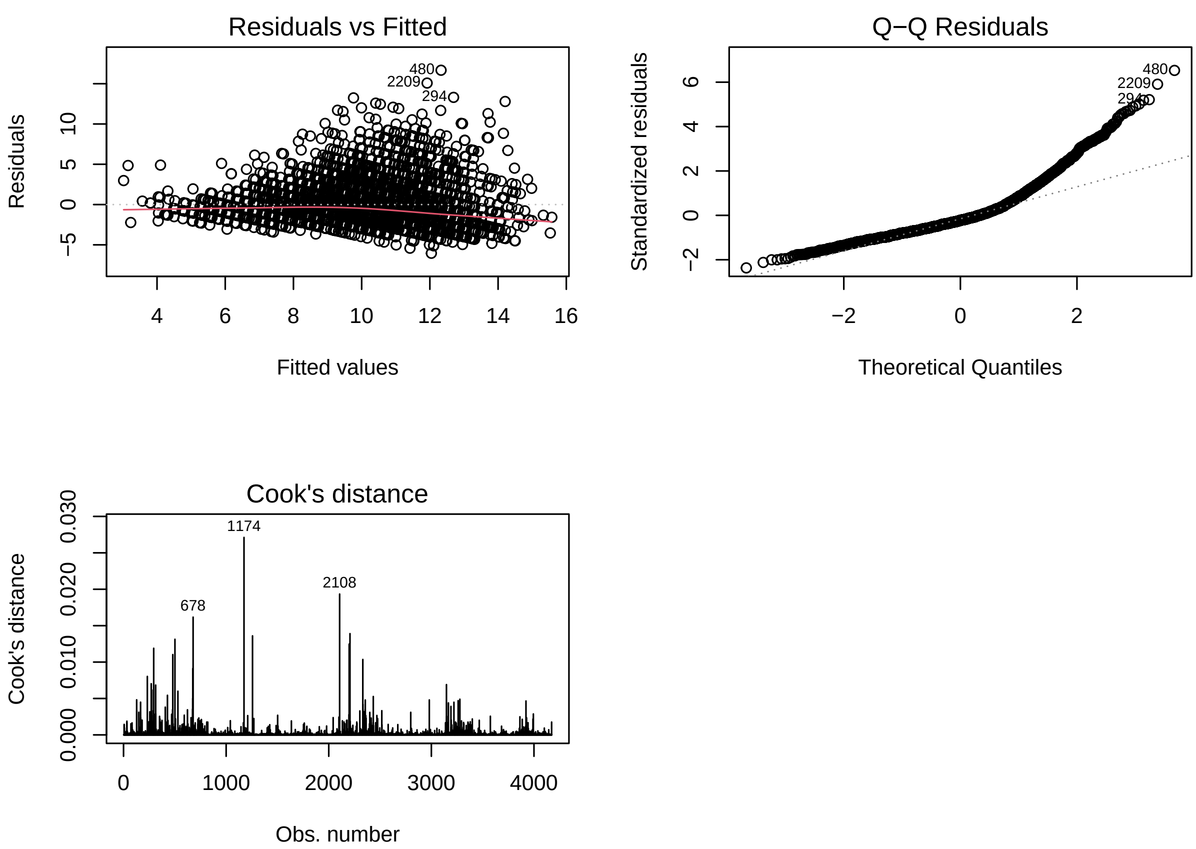
(c) Suppose two abalones are equal in length, but the height measurement of one is 1 unit greater than that of the other. What does the fitted model say about the difference in their numbers of rings?

(d) Give the value of the ratio $\text{SS}_{\text{Error}} \,/\, \text{SS}_{\text{Tot}}$.

(e) How many abalones are in the data set?

(f) Give the p value for testing whether the slope coefficient for the length covariate is equal to zero. Interpret this value.

```r
par(mfrow=c(2,2))
plot(lm1,which=1)
plot(lm1,which=2)
plot(lm1,which=4)
```



(g) Comment on whether you think there are any outliers in this data set.

(h) State whether you think the residuals versus fitted values plot reveals anything that should make us cautious about making inferences with the multiple linear regression model.

(i) State whether you think the normal quantile-quantile plot reveals anything that should make us cautious about making inferences with the multiple linear regression model.

(j) Use numbers given in the output to compute the value which will be returned by `pt(1.498,4171)`