**NAME: KEY**

**Stat 205 Exam I**

There are four problems total. Each lettered part of a problem is worth 6 points; there are 120 points total. However, your exam grade is out of 100 points, so there are 20 points of possible extra credit for this exam. Write clearly and show steps for partial credit. This exam is open book.

---

1. The time to completion of the Rome marathon in 1996 is approximately normal with mean $\mu = 230$ minutes and standard deviation $\sigma = 36$ minutes. Let $Y$ be the time from a randomly selected runner at this race.

   (a) What is the probability of a randomly selected runner completing the race in under three hours, $\Pr\{Y < 180\}$?
   **Answer**:

   $$\begin{aligned} \Pr\{Y < 180\} &= \Pr\left\{\frac{Y - 230}{36} < \frac{180 - 230}{36}\right\} \\ &= \Pr\{Z < -1.39\} \\ &= 0.0823. \end{aligned}$$

   (b) What is the probability of a randomly selected runner completing the race within three to four hours, $\Pr\{180 < Y < 240\}$?
   **Answer**:

   $$\begin{aligned} \Pr\{180 < Y < 240\} &= \Pr\left\{\frac{180 - 230}{36} < \frac{Y - 230}{36} < \frac{240 - 230}{36}\right\} \\ &= \Pr\{-1.39 < Z < 0.28\} \\ &= \Pr\{Z < 0.28\} - \Pr\{Z < -1.39\} \\ &= 0.6103 - 0.0823 \\ &= 0.528 \end{aligned}$$

   (c) What is the probability of a randomly selected runner completing the race with a time over five hours?
   **Answer**:

   $$\begin{aligned} \Pr\{Y > 300\} &= \Pr\left\{\frac{Y - 230}{36} > \frac{300 - 230}{36}\right\} \\ &= \Pr\{Z > 1.94\} \\ &= 1 - \Pr\{Z < 1.94\} \\ &= 1 - 0.9738 \\ &= 0.0262 \end{aligned}$$

   (d) What is the percentile $y^*$ such that 10% of the racers have times below $y^*$, i.e. $\Pr\{Y < y^*\} = 0.1$?
   **Answer**: Using the normal table "in reverse" we find $\Pr\{Z < -1.28\} = 0.1003$, which is as close as we can get to $0.1$. Then

   $$y^* = \mu + \sigma z^* = 230 + 36(-1.28) = 183.9 \text{ minutes.}$$

   (e) Draw a picture of the $N(230, 36^2)$ (normal with mean 230 minutes and standard deviation 36 minutes) density for race times:

2. In a study of the effects of smoking, 9793 pregnant women were asked about their smoking habits. The following classifies each by whether their infant was low birthweight (less than 2500 grams) or normal birthweight, and whether they smoked:

| | Smoking status | | |
|---|---|---|---|
| Birthweight | Smoker | Nonsmoker | Total |
| Low | 237 | 197 | 434 |
| Normal | 3489 | 5870 | 9359 |
| Total | 3726 | 6067 | 9793 |

(a) What is the probability of low birthweight?
**Answer**:
$$\frac{434}{9793} = 0.044.$$

(b) What is the probability of low birthweight among smokers? That is, given that a woman smokes, what is the probability of low birthweight?
**Answer**:
$$\frac{237}{3726} = 0.064.$$

(c) What is the probability of low birthweight among nonsmokers??
**Answer**:
$$\frac{197}{6067} = 0.032.$$

(d) Which is more likely to result in *normal* birthweight, smoking or nonsmoking?
**Answer**: Nonsmoking, because Pr{low|smoker} $>$ Pr{low|nonsmoker}, i.e. $0.064 > 0.032$.

(e) Is *smoking* independent of *birthweight*? Why or why not?
**Answer**: No, Pr{low|smoker} $\neq$ Pr{low}, i.e. $0.064 \neq 0.044$.

3. A certain drug cures 90% of the cases of hookworm in children. Suppose that 5 children suffering from hookworm are to be treated, and that the children can be regarded as a random sample from the population. Let $Y$ be the number of children out of 5 that are cured.

   (a) What kind of random variable is $Y$? Be precise.
   **Answer**: $Y \sim$ binomial$(5, 0.9)$.

   (b) What is Pr$\{Y = 20\}$, the probability that all 5 children will be cured?
   **Answer**:

$$\Pr\{Y = 20\} = {}_5C_5 0.9^5 0.1^0 = 0.9^5 = 0.590.$$

   (c) What is Pr$\{Y = 0\}$, the probability that none of the children will be cured?
   **Answer**:

$$\Pr\{Y = 0\} = {}_5C_0 0.9^0 0.1^{20} = 0.1^5 = 0.00001$$

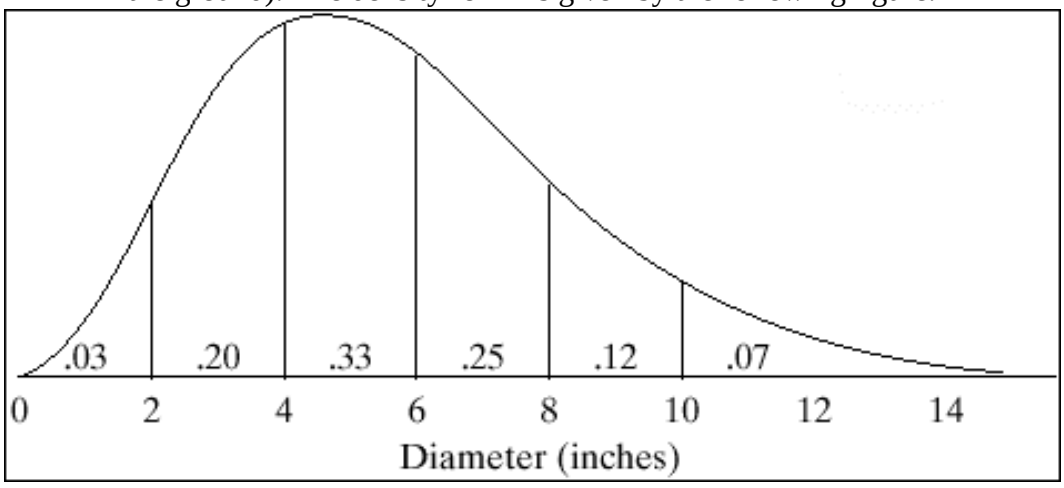
   (d) What the probability that two or fewer of the children will be cured?
   **Answer**:

$$\begin{aligned} \Pr\{Y \leq 2\} &= \Pr\{Y = 0\} + \Pr\{Y = 1\} + \Pr\{Y = 2\} \\ &= {}_5C_0 0.9^0 0.1^5 + {}_5C_1 0.9^1 0.1^4 + {}_5C_2 0.9^2 0.1^3 \\ &= (1)0.1^5 + (5)0.9^1 0.1^4 + (10)0.9^2 0.1^3 \\ &= 0.00856. \end{aligned}$$

   (e) What is the mean and *standard deviation* of $Y$?
   **Answer**:

$$\mu_Y = np = 5(0.9) = 4.5, \quad \sigma_Y = \sqrt{np(1-p)} = \sqrt{5(0.9)(0.1)} = 0.671.$$

4. Let $Y$ be the diameter (in inches) of a randomly selected tree trunk (measured 4.5 feet off the ground). The density for $Y$ is given by the following figure.




(a) What is Pr$\{Y > 4\}$?
**Answer**: Formally,

$$\begin{aligned} \Pr\{Y > 4\} &= \Pr\{4 < Y < 6\} + \Pr\{6 < Y < 8\} + \Pr\{8 < Y < 10\} + \Pr\{Y > 10\} \\ &= 0.33 + 0.25 + 0.12 + 0.07 \\ &= 0.77. \end{aligned}$$

(b) What is Pr$\{4 < Y < 8\}$?
**Answer**:

$$0.33 + 0.25 = 0.58.$$

(c) Is Pr$\{Y > 12\}$ larger or smaller than 0.07?
**Answer**: Smaller, because this area under the curve is less than 0.07.

(d) Find the $56$th percentile, i.e. the $y^*$ such that Pr$\{Y < y^*\} = 0.56$.
**Answer**: Since Pr$\{Y < 6\} = 0.56$, the $56$th percentile is $y^* = 6$ inches.

(e) Is this density skewed right, skewed left, or symmetric? Is this density unimodal or multimodal?
**Answer**: The density is skewed right and unimodal.