NAME:

Stat 205 Quiz 3

Let $D+$ denote that an individual is infected with Hepatitis C and $D-$ denotes an individual is disease-free. González et al. (2008) discuss a test for Hepatitis C that has sensitivity $\Pr\{T+|D+\} = 0.867$ and perfect specificity $\Pr\{T-|D-\} = 1.000$. The prevalence of Hepatitis C is $\Pr\{D+\} = 0.0074$ in the general population. Here are some general rules for *any* events A and B:

- $\Pr\{A\} = \Pr\{A|B\}\Pr\{B\} + \Pr\{A|B^C\}\Pr\{B^C\}$ (law of total probability).
- $\Pr\{B|A\} = \dfrac{\Pr\{A|B\}\Pr\{B\}}{\Pr\{A\}}$ (Bayes' rule).
- $\Pr\{A^C|B\} = 1 - \Pr\{A|B\}$ (compliment rule for conditional probability).

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1. Find the probability that a test comes up positive $\Pr\{T+\}$.
   Answer: Use the law of total probability and the complement rules

$$\begin{aligned}\Pr\{T+\} &= \Pr\{T+|D+\}\Pr\{D+\} + \Pr\{T+|D-\}\Pr\{D-\} \\ &= \Pr\{T+|D+\}\Pr\{D+\} + (1 - \Pr\{T-|D-\})(1 - \Pr\{D+\}) \\ &= 0.867 \times 0.0074 + (1-1) \times (1 - 0.0074) \\ &= 0.00642.\end{aligned}$$

2. Find the probability of having the disease given the test comes up positive $\Pr\{D+|T+\}$.
   Answer: Use Bayes' rule

$$\begin{aligned}\Pr\{D+|T+\} &= \frac{\Pr\{T+|D+\}\Pr\{D+\}}{\Pr\{T+\}} \\ &= \frac{0.867 \times 0.0074}{0.00642} \\ &= 1.\end{aligned}$$

3. Is $D+$ independent of $T+$? Why or why not?
   Answer: No, they are dependent because $\Pr\{D+|T+\} = 1$ which is different from $\Pr\{D+\} = 0.0074$. Knowing that the test came back positive $T+$ changes the probability of having Hepatitis C from 0.0074 to 1.