Stat 205 Quiz 8

To investigate the dependence of energy expenditure on body build, researchers used underwater weight techniques to determine the fat-free body mass for each of seven men. The also measured the total 24-hour energy expenditure for each man during conditions of quiet sedentary activity. Let Y be the energy expenditure (kcal) and X be the fat-free mass (kg). The data are plotted here:



Here are the statistics needed to answer the questions below: $\bar{x} = 62.40, \ \bar{y} = 2,168,$ $\sum (x_i - \bar{x})^2 = 877.74, \quad \sum (x_i - \bar{x})(y_i - \bar{y}) = 21,953.7, \quad SS(resid) = 21,026.1.$

(a) Compute b_0 and b_1 from regressing Y on X. What is the fitted regression line? Answer:

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{21,953.7}{877.74} = \boxed{25.01}$$

$$b_0 = 2,168 - 25.012(62.40) = \boxed{607.3}$$

The fitted line is

$$Y = 607.3 + 25.01 x$$

- (b) Find a 95% confidence interval for β_1 .
 - Answer: First, $s_{y|x} = \sqrt{SS(resid)/(n-2)} = \sqrt{21,026.1/(7-2)} = 64.85$. Then $SE_{b_1} = s_{y|x}/\sqrt{\sum(x_i \bar{x})^2} = 64.85/\sqrt{877.74} = 2.19$. From the back of the book, $t_{0.975} = 2.571$ on df = 7 - 2 = 5. So a 95% confidence interval is $25.01 \pm 2.571(2.19)$ giving (19.4,30.6)

- (c) Test $H_0: \beta_1 = 0$ at the 5% significance level. What does this test imply about the relationship between fat-free mass and energy expenditure? Answer: Since the 95% confidence interval does not include zero, we reject $H_0: \beta_1 = 0$ at the 5% significance level. There is a significant (positive) linear relationship between energy expenditure and fat-free mass.
- (d) Answer: At X = 65 kg, we have

 $\hat{Y} = 607.3 + 25.01 \times 65 \approx$ 2233 kcal