

Multinomial Experiment.  $\rightarrow$  Indep. identical trials, each result falls into one of  $k$  categories

Parameters to be estimated:  $p_1, p_2, \dots, p_k$

Multinomial likelihood based on  $\underline{x} = (x_1, x_2, \dots, x_k)$  out of  $n$  trials:

$$L(p_1, \dots, p_k | x_1, \dots, x_k) = \frac{n!}{x_1! \dots x_k!} p_1^{x_1} \dots p_k^{x_k}$$

Dirichlet  $(\alpha_1, \alpha_2, \dots, \alpha_k)$  prior:

$$p(p_1, \dots, p_k) = \frac{\Gamma(\alpha_1 + \dots + \alpha_k)}{\Gamma(\alpha_1) \dots \Gamma(\alpha_k)} p_1^{\alpha_1 - 1} \dots p_k^{\alpha_k - 1}$$

Posterior  $\propto L(p_1, \dots, p_k | \underline{x}) p(p_1, \dots, p_k)$

$$\begin{aligned} \pi(p_1, \dots, p_k | \underline{x}) &\propto p_1^{x_1} \dots p_k^{x_k} p_1^{\alpha_1 - 1} \dots p_k^{\alpha_k - 1} \\ &= p_1^{x_1 + \alpha_1 - 1} \dots p_k^{x_k + \alpha_k - 1} \end{aligned}$$

which is Dirichlet  $(x_1 + \alpha_1, \dots, x_k + \alpha_k)$

$\Rightarrow$  Conjugate!