The linear model for the Latin square is:

$$y_{ijk} = \mu + \rho_i + \gamma_j + \tau_k + \varepsilon_{ijk},$$

where

y_{ijk} = observed response of treatment k in row i and column j,

μ = reference value or overall mean,

ρ_i = effect of row $i, i = 1, 2, \ldots, t$,

γ_j = effect of column $j, j = 1, 2, \ldots, t$,

τ_k = effect of treatment $k, k = 1, 2, \ldots, t$, and

ε_{ijk} = random error.

As before, the blocking effects are usually considered random with means zero and variance σ_ρ^2 and σ_γ^2, respectively, and we add the restriction $\Sigma\tau_i = 0$ for the fixed treatment effects. Blocking effects may be fixed, but the outline of the analysis is not changed. There are no interaction terms in the model because interactions between row, column, or treatment effects constitute a violation of assumptions underlying the use of this design. Since violations of this assumption are difficult to detect, care must be taken to use this design only when such an interaction is not expected to exist.

The partitioning of sums of squares for the analysis of variance is relatively straightforward. The row, column, and treatment sums of squares are computed using the respective means and the error sum of squares is obtained by subtraction. The table of expected mean squares (which is not reproduced here) shows that the error mean square obtained in this manner is indeed the proper denominator for F ratios, assuming the assumption of no interaction effects holds.

This example of a Latin square design concerns a hypothetical experiment on the effect of various types of background music on the productivity of workers in a plant. It is well known that productivity differs among the various days of the week as well as during different times of day. Hence we design the experiment as a Latin Square with hours of the day as rows and days of the week as columns. There are five music "treatments":

A: rock and roll
B: country/western
C: easy listening
D: classical
E: no music.

The row blocks are five one-hour periods and the column blocks are the five working days. Note that these blocking factors are indeed fixed. The response is the number of parts produced. The data are given in Table 10.7 where the indicator for treatment is given under each response value. Note that the presentation of the data does not show the randomization.

TABLE 10.7

DATA FOR LATIN SQUARE DESIGN

Times	Mo	Tu	Day We	Th	Fr	Means
9–10	6.3 (A)	9.8 (B)	14.3 (C)	12.3 (D)	9.1 (E)	10.36
10–11	7.7 (B)	13.5 (C)	13.4 (D)	12.6 (E)	9.9 (A)	11.42
11–12	11.7 (C)	10.7 (D)	13.8 (E)	9.0 (A)	10.3 (B)	11.10
1–2	9.0 (D)	10.5 (E)	9.3 (A)	9.8 (B)	12.0 (C)	10.12
2–3	4.5 (E)	5.3 (A)	8.4 (B)	9.6 (C)	11.0 (D)	7.76
Means	7.84	9.96	11.84	10.66	10.46	(overall) 10.15
Treatment Means	A 7.96	B 9.20	C 12.22	D 11.28	E 10.10	

The computations for the analysis of variance are performed as if each factor (rows, columns, and treatments) are main effects. The error sum of squares is obtained by subtraction of all factor sums of squares from the total sum of squares. For this example:

$$\begin{aligned}
\text{TSS} &= (6.3 - 10.15)^2 + (9.8 - 10.15)^2 + \cdots + (11.0 - 10.15)^2 \\
&= 154.362 \\
\text{SS(Times)} &= 5[(10.36 - 10.15)^2 + (11.42 - 10.15)^2 + \cdots + (7.76 - 10.15)^2] \\
&= 41.362 \\
\text{SS(Days)} &= 5[(7.84 - 10.15)^2 + (9.96 - 10.15)^2 + \cdots + (10.46 - 10.15)^2] \\
&= 56.314 \\
\text{SS(Music)} &= 5[(7.96 - 10.15)^2 + (9.20 - 10.15)^2 + \cdots + (10.46 - 10.15)^2] \\
&= 56.314.
\end{aligned}$$

The error sum of squares is obtained by subtraction:

$$\begin{aligned}
\text{SSE} &= \text{TSS} - \text{SS(Times)} - \text{SS(Days)} - \text{SS(Music)} \\
&= 13.763.
\end{aligned}$$

The degrees of freedom are 4 for each of the factors, and, by subtraction, 12 for error.

The results are summarized in the usual analysis of variance format in Table 10.8. The F values show that the hypotheses of no treatment, row, or column effect are all rejected ($\alpha < 0.05$). When ranked from high to low, the treatment means are almost equally spaced. In this case post hoc paired comparisons may be appropriate, unless there is some prior information on