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Name: Key

STAT 516 – Test 2 – Fall 2026

1. When selecting variables, we would like a model with an adjusted R^2 that is:

(A) as high as possible (B) equal to SSR / TSS (C) as low as possible (D) near $p + 1$

2. A multiple regression analysis was performed in which a sample of 34 observations was used to fit the model. There were 8 candidate predictor variables under consideration. Consider the following candidate models (all of these models include an intercept term):

Full Model (with 8 predictor variables) has $R^2 = 0.7824$
Medium Model (with 6 predictor variables) has $R^2 = 0.7717$
Small Model (with 5 predictor variables) has $R^2 = 0.7191$

(a) The Mallows' Cp for the Small Model is 10.28, and the Cp for the Medium Model is 6.23. Explain carefully what this indicates about the respective quality of these two models.

Small model has $C_p = 10.28 >> 6$ and Medium has $C_p = 6.23 \approx 7$, so the medium model has less bias relative to the full model.

(b) What is the numerical value of the Cp for the Full Model? 9

(c) A researcher said, "I believe the Full Model is the preferred model since its R^2 value is the highest of the three candidate models." Do you agree or disagree? Briefly explain.

Disagree – since R^2 always increases when we add variables, cannot use R^2 to compare models with different number of variables.

3. Consider a regression analysis in which two variables were measured on 48 states: infant mortality (Y), and teenage birth rate (X). We believe the two variables may be related. A simple linear regression of Y on X was conducted. The DFFITS values for this regression were calculated, and the highest DFFITS in absolute value was 0.690, for South Dakota. Is there evidence that any of the observations has an undue influence on the regression line? Explain, *justifying your answer mathematically*.

$n = 48$, $m+1 = 2$

$2\sqrt{\frac{m+1}{n}} = 2\sqrt{\frac{2}{48}} = 0.408$

Yes – SD is influential, having $|DFFITS| = 0.690 > 0.408$

4. In a one-way ANOVA, the residuals are formed by subtracting: each response value minus the

(A) sample mean for that value's level (B) sample variance for that value's level
(C) grand mean (D) population mean for that value's level

Version B Answers: 14 46.0 4.0 78.2

5. For this one-way ANOVA, can you fill in the four blank values in the following ANOVA table?

Source	df	SS	MS	F*
Between	2	18.4	9.20	2.875
Within	12	38.40	3.20	
Total	14	56.8		

6. An experiment was conducted to determine the effectiveness of sleep-inducing drugs. An equal number of insomniacs was assigned to each of three different treatment groups. The groups were: 1 = placebo (fake drug); 2 = standard drug; 3 = experimental drug. The number of hours per night of sleep was observed for each subject in the study over a fixed period of time. The ANOVA table is given here:

```
Response: sleephrs
           Df Sum Sq Mean Sq F value
drugtype    2 33.163 16.5817  15.041
Residuals 15 16.537  1.1024
```

(a) Based on the ANOVA table, how many subjects (insomniacs) were in each group? How do you know? Total df = 17, so there were 18 subjects overall, so 6 in each group.

(b) Assume for now that the usual ANOVA model assumptions are met. Conduct an formal test to determine whether the mean nightly sleep amounts differs across the three groups. State the null and alternative hypotheses, give the test statistic value and the appropriate table value (using $\alpha = 0.05$), and state your conclusion.

$H_0: \mu_1 = \mu_2 = \mu_3$ H_a: not all μ_i's are the same

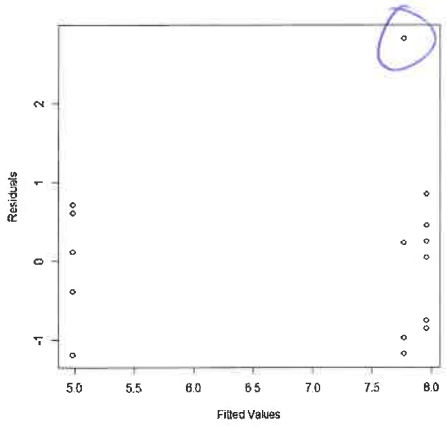
$F^* = 15.041$ $F_{.05,2,15} = 3.68$

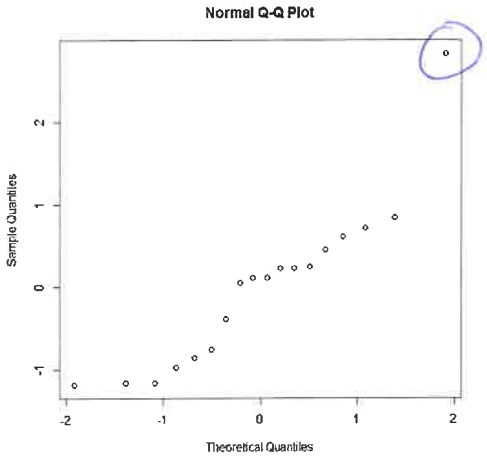
Since $15.041 > 3.68$, reject H_0 and conclude the mean nightly sleep amount differs across the three groups.

(c) A residual plot and a normal Q-Q plot of the residuals are shown at the top of the next page. Are there any outliers in the data set? If so, circle the outlier(s) on the plot.

Yes, one outlier.

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(d) State the two model assumptions that we can assess using these two plots, and briefly explain what these plots here tell you about whether these model assumptions are satisfied or violated.

equal variances — mostly satisfied except one outlier

normality of errors — mostly satisfied except one outlier

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(e) A Levene test was done for this ANOVA and the following output was obtained in R. Briefly, what is your conclusion from this output?

```
> Levene.test(sleephrs, drugtype)
          Df F value Pr(>F)
group      2   1.656  0.224
Residuals 15
```

- Since p-value = 0.224 > 0.05, we conclude equal variances across the 3 groups.

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(f) Suppose that we didn't have any preplanned comparisons in mind, so we did a Tukey multiple comparisons test. The resulting output in R was:

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = t2.fit)

$drugtype
          diff       lwr      upr     p adj
2-1  2.9666667  1.392074 4.541260 0.0005375
3-1  2.7833333  1.208740 4.357926 0.0009681
3-2 -0.1833333 -1.757926 1.391260 0.9509947
```

Only the standard drug (2) and the experimental drug (3) are NOT significantly different

What are your conclusions about which pairs of groups ~~have~~ DO NOT have significantly different mean nightly sleep hours? If you were an insomniac, would you risk taking the new experimental drug rather than the standard drug? ~~The standard drug and the placebo are significantly different. The experimental drug and the placebo are significantly different.~~ There is no need to risk the experimental drug, since it's not significantly different from the standard drug.

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(f) Based on the output in part (f) and the plots in part (c), what is the *approximate* mean nightly hours of sleep for subjects in group 1?

Around 5 hours

3

(g) If we had actually wanted to compare the mean nightly sleep of the placebo to the mean for the actual drugs, write the contrast that we could use to formally compare whether these means differ.

$L = \mu_1 - \frac{1}{2}\mu_2 - \frac{1}{2}\mu_3$

4

7. With a one-way ANOVA, in what situation would using a multiple comparison procedure be appropriate? (choose one)

(A) When the overall ANOVA F-test is not significant and we have no pre-planned comparison among levels in mind.
(B) When the overall ANOVA F-test is not significant and we have a pre-planned comparison among levels in mind.
(C) When the overall ANOVA F-test is significant and we have no pre-planned comparison among levels in mind.
(D) When the overall ANOVA F-test is significant and we have a pre-planned comparison among levels in mind.

3

8. Consider the two-way ANOVA model $Y_{ijk} = \mu + \alpha_i + \gamma_j + (\alpha\gamma)_{ij} + \varepsilon_{ijk}$, where $i = 1, 2, 3$. (That is, factor A has three levels.) Suppose $\alpha_1 = 2.5$. Then which of the following is possible? [Hint: What should the α_i's sum to?]

(A) $\alpha_2 = 0, \alpha_3 = 0$
(B) $\alpha_2 = -1.5, \alpha_3 = -1$
(C) $\alpha_2 = 2.5, \alpha_3 = 2.5$
(D) $\alpha_2 = -7.5, \alpha_3 = 2.5, \alpha_4 = 2.5$

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9. A factorial experiment was done to analyze the range of rockets (in miles traveled) when they were launched using: (factor A) several different types of launcher and (factor C) several different kinds of fuel. Some R output from the analysis is given below

Analysis of Variance Table

```
Response: miles
              Df Sum Sq Mean Sq F value     Pr(>F)
launcher       2 38.203  19.102  27.386 0.00003 ***
fuel           3 33.548  11.183  16.033  0.0002 ***
launcher:fuel  6  7.797   1.299   1.863  0.1691
Residuals     12  8.370   0.697
```

(a) From the ANOVA table, we see this represents a _____ factorial experiment. (Choose the best answer)

(A) 2 × 3 (B) 3 × 6 (C) 2 × 6 (D) 3 × 4 (E) 2 × 3 × 6

3

(b) Perform the F-test for interaction between launcher type and fuel type. Give the test statistic value, the P-value, and your conclusion. (Use $\alpha = 0.05$.)

$F^* = 1.863$, P-value $= 0.1691 > .05$

-We conclude there is no significant interaction.

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(c) In a sentence, state what it would mean, *in the context of this data set*, to have interaction between the two factors. The effect of launcher type on rocket range depends on the type of fuel used.

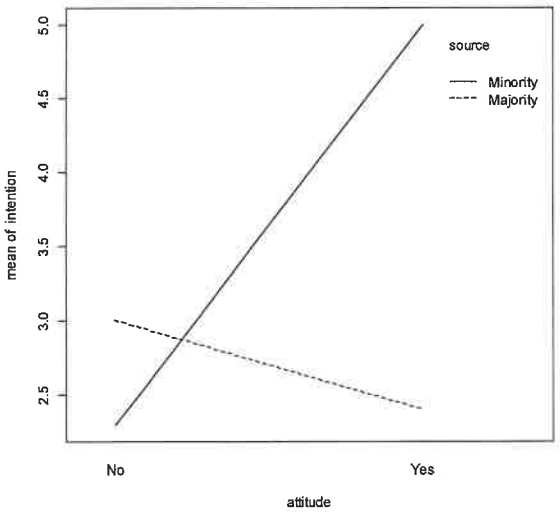
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(d) In the two-way ANOVA, we may conduct F-tests for main effects if and only if

(A) there is significant interaction.
(B) the study has balanced data.
(C) there is no significant interaction.
(D) the MSW is an unbiased estimator of the error variance.

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10. An experiment was conducted with a response variable called "intention" and two factors ("Source" and "Attitude"), each having two levels. A profile plot of the sample means at each factor level combination is given below. Based on this profile plot, is there *interaction* between the two factors? *Briefly* explain why or why not.



Yes — the profiles are not parallel. The effect of attitude on intention is different for the two sources.

4

Extra Credit 1: The Fisher LSD procedure stands for "Least Significant Difference". What does the HSD is Tukey's HSD procedure stand for?

Honestly Significant Difference

+2

Extra Credit 2: All things being equal, which procedure has more POWER to reject null hypotheses, Fisher's or Tukey's?

Fisher's.

+2