APPENDIX 2

Common Probability Distributions, Means, Variances, and Moment-Generating Functions

Table 1 Discrete Distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Binomial	$p(y) = \binom{n}{y} p^y(1-p)^{n-y};$ $y = 0, 1, \ldots, n$	np	$np(1-p)$	$[pe^t + (1-p)]^n$
Geometric	$p(y) = p(1-p)^{y-1};$ $y = 1, 2, \ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Hypergeometric	$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}};$ $y = 0, 1, \ldots, n$ if $n \le r$, $y = 0, 1, \ldots, r$ if $n > r$	$\frac{nr}{N}$	$n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$	does not exist in closed form
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!};$ $y = 0, 1, 2, \ldots$	λ	λ	$\exp[\lambda(e^t - 1)]$
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r(1-p)^{y-r};$ $y = r, r+1, \ldots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$

Table 2 Continuous Distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Uniform	$f(y) = \frac{1}{\theta_2 - \theta_1}; \theta_1 \le y \le \theta_2$	$\frac{\theta_1 + \theta_2}{2}$	$\frac{(\theta_2 - \theta_1)^2}{12}$	$\frac{e^{t\theta_2} - e^{t\theta_1}}{t(\theta_2 - \theta_1)}$
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2\right]$ $-\infty < y < +\infty$	μ	σ^2	$\exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$
Exponential	$f(y) = \frac{1}{\beta} e^{-y/\beta}; \quad \beta > 0$ $0 < y < \infty$	β	β^2	$(1 - \beta t)^{-1}$
Gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta};$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$
Chi-square	$f(y) = \frac{(y)^{(\nu/2)-1} e^{-y/2}}{2^{\nu/2}\Gamma(\nu/2)};$ $y > 0$	ν	2ν	$(1 - 2t)^{-\nu/2}$
Beta	$f(y) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1}(1-y)^{\beta-1};$ $0 < y < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	does not exist in closed form

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Additional Formulas

If $Y \sim \text{beta}(\alpha, \beta)$ with α, β both integers:

$$F(y) = P(Y \leq y) = \sum_{i=\alpha}^{\alpha+\beta-1} \binom{\alpha+\beta-1}{i} y^i (1-y)^{(\alpha+\beta-1)-i}$$

[a sum of $\text{Bin}(n = \alpha+\beta-1, p = y)$ probabilities]

For $k > 0$:

$$P\{|Y-\mu| < k\sigma\} \geq 1 - \frac{1}{k^2} \Leftrightarrow P\{|Y-\mu| \geq k\sigma\} \leq \frac{1}{k^2}$$

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Multinomial Probability Function:

$$p(y_1, y_2, \ldots, y_k) = \frac{n!}{y_1!\, y_2! \cdots y_k!}\, p_1^{y_1} p_2^{y_2} \cdots p_k^{y_k}$$

If (Y_1, Y_2) bivariate normal:

$$Y_1 \mid Y_2 = y_2 \sim N\left[\mu_1 + \rho\left(\frac{\sigma_1}{\sigma_2}\right)(y_2 - \mu_2),\ \sigma_1^2(1-\rho^2)\right]$$

$$Y_2 \mid Y_1 = y_1 \sim N\left[\mu_2 + \rho\left(\frac{\sigma_2}{\sigma_1}\right)(y_1 - \mu_1),\ \sigma_2^2(1-\rho^2)\right]$$