STAT 521 – Spring 2019 – HW 1

Note: You must show your work wherever possible!

1. Suppose three balanced 6-sided dice are thrown. What is the probability that the same number appears on exactly two of the three dice?

2. Suppose 5 percent of men are colorblind and 0.25 percent of women are colorblind. A colorblind person is chosen at random from the population. What is the probability that this person is male? (Assume that there are an equal number of males and females in the population.)

3. On a high school debate team there are four freshman boys, six freshman girls, and six sophomore boys, along with an unspecified number of sophomore girls. Consider selecting a student at random from the debate team and recording the sex and the class (year in school) of the selected student. How many sophomore girls must be on the team if sex and class are to be independent when a student is selected at random?

4. Suppose one basket has five white balls and seven black balls. A second basket has three white balls and twelve black balls. We will flip a fair coin and if it lands "heads" a ball from basket 1 is selected, and if it lands "tails" a ball from basket 2 is selected. Suppose that we have done this and that a white ball was selected. What is the probability that the coin landed tails?

5. Suppose we toss two fair coins. Let X_1 be the number of coins landing heads and X_2 be the number of coins landing tails. Let $X = X_1 - X_2$. List the possible values that X could take, and give the probabilities associated with each of these values.

6. Suppose two teams are playing a series of games, each of which is independent. Team A has probability p of winning each game, and team B has probability $1-p$ of winning each game. The winner of the series is the first team to win two games. Find the expected number of games played; note that this will be a function of p. [Hint: Let X be the total number of games played in the series and first determine the probability for each possible value of X.] Show that this expected number of games played is maximized when $p = 1/2$.

7. Consider an urn with $n + m$ balls, of which n are red and m are black. We remove the balls from the urn at random one at a time, without replacement. Let X be the number of red balls removed before the first black ball is chosen. We want to find $E(X)$.

   Number the red balls from 1 to n and define the random variables $X_i, i = 1, \ldots, n$ by: $X_i = 1$ if the red ball labeled i is removed before any black ball is chosen, and $X_i = 0$ otherwise.

   (a) What is a simpler way to express $\sum_{i=1}^{n} X_i$?

   (b) Find $E(X)$. [Hint: Since X_i is a random variable taking only the values 0 and 1, what is $E(X_i)$?]

8. A manuscript is sent to a typing firm consisting of typists A, B, and C. If it is typed by A, then the number of errors made is a Poisson random variable with mean 2.6; if it is typed by B, then the number of errors made is a Poisson random variable with mean 3.0; if it is typed by C, then the number of errors made is a Poisson random variable with mean 3.4. Assume that each typist in the firm is equally likely to do the work. Let X denote the number of errors in the typed manuscript.

   Find both $E(X)$ and $Var(X)$. [Hint: Define a random variable Y that takes three possible values, and then use the "iterated" formulas.]

9. Suppose Y is Uniform(0,1) and the conditional distribution of $X|Y$ is Uniform$(0, Y)$. Find $E(X)$ and $Var(X)$.

   Graduate Students Must Also Do TWO of The Following Problems (Undergraduates may do any of these for extra credit):

10. Assume that each child born is equally likely to be a boy or a girl.
    (a) If a family has two children, what is the probability that both are girls, given that the oldest is a girl?
    (b) If a family has two children, what is the probability that both are girls, given that at least one is a girl?

11. An urn contain five red balls, three orange balls, and two blue balls. Two balls are randomly and simultaneously selected. Let X be the number of orange balls selected. What are the possible values of X? Calculate $P[X = 0]$. [Hint: How many ways are there to select 2 balls from the 10 balls? How many ways are there to select two balls from the set of non-orange balls?]

12. Let $P[X = 0] = p_0$, where $0 < p_0 < 1$. Let $E(X) = \mu$ and $Var(X) = \sigma^2$.
    (a) Find $E(X|X \neq 0)$. [Hint: Condition on whether or not X equals 0.]
    (b) Find $Var(X|X \neq 0)$.
    Hint: Note $Var(X|X \neq 0) = E(X^2|X \neq 0) - [E(X|X \neq 0)]^2$.

13. **This is an excellent example of finding probabilities and expected values by conditioning.** Suppose A and B play a series of games with A winning each game with probability p. The overall winner of the series is the first player to have won two more games than the other.
    (a) Find the probability the A is the overall winner of the series.
    (b) Find the expected number of games played.
    Hint: Let A = the event that "A wins". Let X = the overall number of games played in the series. Let Y = the number of games won by A out of the first two games.