STAT 509 --- Section 3.1: Probability

Basic Definitions

Sample Space (of an experiment): The collection of all the possible outcomes (or sample points).

Example 1. Record plant location (either 1, 2, 3) of next maintenance call for spinning machine repair:
Sample space =

Example 2. Toss 2 fair coins:
Sample space =

Example 3. An engineering design firm is up for a Nissan contract and a Ford contract:
Sample space =

The probability of a sample point is a number between 0 and 1 that measures the likelihood that this outcome will occur when the experiment is performed.

Often we take this to mean the proportion of times the outcome would occur if we repeated the experiment many times.

Note:
(1) All sample point probabilities must be between 0 and 1.
(2) The probabilities of all the points in the sample space must sum to 1.

Example 1: Plant 1 has 22 machines, plant 2 has 60 machines, plant 3 has 18 machines. Probability of next repair being from plant 3, denoted:

P(3) =

Assumption?

An <u>event</u> is an outcome or collection of outcomes.

We typically determine the probability of an event by adding the probabilities of the distinct outcomes that make up the event.

Example 1: Event A = 'next repair from odd-numbered plant'

P(A) =

Example 3: Event B = 'get at least one contract'

P(B) =

<u>Unions and Intersections</u>

<u>Compound events</u> are composed of two or more "simple events," for example:

- **The <u>union</u> of events A and B is the event that either A <u>or</u> B (or both) occurs.**
- **Denoted A ∪ B**

- The <u>intersection</u> of events A and B is the event that both A and B occur when the experiment is conducted.
- Denoted $A \cap B$

<u>Venn Diagrams</u>: Represent graphically which outcomes make up which events.

Pictures:

<u>Mutually Exclusive Events</u>

Two or more events are <u>mutually exclusive</u> when the following is true: If one event occurs in an experiment, the other event cannot occur.

<u>Note</u>: Formally speaking, two events A and B are mutually exclusive if $P(A \cap B) = 0$. (That is, A and B have no outcomes in common.)

Example 4: A manufacturer (Acme) of automobile lamps categorizes the lamps produced as "Good", "Satisfactory", or "Unsatisfactory" for both *intensity* and *useful life*. For 200 lamps, consider the table:

		Useful Life		
Intensity	**Good**	**Satis**	**Unsatis**	**Total**
Good	**100**	**25**	**5**	
Satis	**35**	**10**	**5**	
Unsatis	**10**	**8**	**2**	
Total				**200**

Select a lamp at random. What is the probability that it will be rated "Good" in *intensity*?
A =
P(A) =

What is the probability that it will be rated "Good" in *useful life*?
B =
P(B) =

What is the probability that it will be rated "Good" in *intensity* and *useful life*?

<u>Probability of a Union of Two Events:</u>

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

What is the probability that it will be rated "Good" in *useful life* or *intensity*?

<u>Complementary Events</u>

The complement of an event A (denoted $\overline{A}$) is the collection of outcomes that do <u>not</u> correspond to A.

Since all outcomes in the sample space are either in A or not in A (and thus are in $\overline{A}$), then:

$$P(A) + P(\overline{A}) = 1$$

Example 2(b): Suppose we toss 10 coins. Define event A = {obtain at least 2 "heads"}.

What is P(A)?

<u>Conditional Probability</u>

$P(A \mid B)$ = probability that event A occurs <u>given that</u> event B occurs when the experiment is conducted.

Recall auto lamp example:
A = {Good in *intensity*}, B = {Good in *useful life*}

What is the probability that the lamp is Good in *intensity* <u>given that</u> the lamp is Good in *useful life*?

Intuitively, note:

Formula: $$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad \text{(assuming } P(B) \neq 0\text{)}$$

Example above:

What is the probability that the lamp is Good in *intensity* <u>given that</u> the lamp is Unsatisfactory in *useful life*?

Rearranging the Conditional Probability Formula, we get:

<u>Probability of Intersection of Two Events</u>

$$P(A \cap B) = P(B)\, P(A \mid B) \quad \text{or} \quad P(A \cap B) = P(A)\, P(B \mid A)$$

Example 4(b): An auto-parts store purchases 30% of its lamps from the Acme manufacturer. Estimate the probability that a randomly chosen lamp in the store is from Acme and is "Unsatisfactory" in *intensity*.

More Examples: When a computer goes down, there is a 75% chance that it is due to an overload and a 15% chance that it is due to a software problem. There is a 15% chance that *neither* an overload nor a software problem is the cause. For a random computer malfunction, what is the probability that both an overload and a software problem are liable?

80% of accidents at a foundry involve human error and 40% involve equipment malfunction. 35% involve both problems. If an accident involves an equipment malfunction, what is the probability that there was also human error?

<u>Independent Events</u>: Events A and B are independent if the occurrence of B doesn't affect the probability that A occurs (and vice versa).

That is, A and B are independent <u>if and only if</u> $P(A \mid B) = P(A)$. We could also say: A and B are independent if and only if
$P(B \mid A) = P(B)$.

Events that are not independent are called <u>dependent</u>.

<u>Intersection of Two Independent Events</u>:

If A and B are independent, then $P(A \cap B) = P(A)\,P(B)$. (Why?)
Conversely, if $P(A \cap B) = P(A)\,P(B)$, then A and B are independent.

<u>Acme example</u>:
A = {Good in *intensity*}, B = {Good in *useful life*}.
Are A and B independent?

Example 6: Roll 1 die. Let A = {even number} and B = {less than 4}. Are A and B independent?

But consider C = {less than 5}. Then A and C <u>are</u> independent. Why?

Note: "Independent" and "Mutually exclusive" are <u>very</u> different concepts.

• If two events A and B are mutually exclusive (and P(A) > 0 and P(B) > 0), then they <u>cannot</u> be independent. Why?

• Many engineering events are independent or can be modeled as independent:

Example: Four electrical components are connected in series. The reliability of each component is 0.90. If the components are independent, what is the probability that the circuit works when the switch is thrown?

• A random sample implies that individual observations are independent of one another.

Law of Total Probability:

P(B) =

Venn Diagram Picture:

Bayes' Rule:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} =$$

• Allows us to express P(A | B) in terms of P(B | A), which may be easier to work with.

Example: Machine 1 makes 30% of all parts in a factory, and Machine 2 makes the rest. 2% of all parts made by Machine 1 are defective, and 3% of all parts made by Machine 2 are defective. Suppose we find a part that is defective. What is the probability that it came from Machine 1?