Randomized Block Design with Sampling

● **Sometimes we may have more than one observation per treatment-block combination**

● **Within each block, we have a sample of $n \geq 2$ observations having the same treatment.**

● **Model equation for RBD with sampling:**

● **ε_{ij} was experimental error → measures variation among units having the same treatment (across the collection of blocks) [var(ε_{ij}) = σ^2]**

● **δ_{ijk} is sampling error → measures variation among units having the same treatment within the same block [var(δ_{ijk}) = σ_δ^2]**

● **In this situation, we must look carefully at the Expected MS to choose the appropriate denominator for our F-statistic.**

● **Assuming treatment effects are fixed and block effects are random:**

Source	df	Expected(MS)

• **Testing for treatment effects:**

Recall H_0:

• **If H_0 is true, then which two Mean Squares have the same expected value?**

• **Appropriate test statistic is:**

$F^* =$ Reject H_0 if:

• **What is the test statistic for testing H_0: $\sigma_\beta^2 = 0$?**

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$F^* =$ Reject H_0 if:

Example: Experiment on stretching ability (Table 10.6, p. 474)

Response = stretching ability of rubber material
Treatments = 7 materials (A, B, C, D, E, F, G)
Blocks = 13 lab sites

● **At each lab, there were $n = 4$ units for each type of material.**

$n = 4$, $t = 7$, $b = 13$ → **total of**
observations overall.

● **Is there a significant difference in mean stretching ability among the seven materials?**

● **We test:**

F* =

Compare to

Software gives P-value:

● **Reject H_0 and conclude there is a significant difference in mean stretching ability among the seven materials.**

● Which of the materials are significantly different in terms of mean stretching ability?

● Can use Tukey multiple comparisons procedure (experimentwise error rate $\alpha = 0.05$).

Results from software:

Latin Square Designs

● Sometimes we may have <u>two</u> blocking factors.

<u>Example</u>: Suppose we are comparing tire performance across four tire brands (label them A, B, C, D).

● The blocking factors are Car (1, 2, 3, 4) and Tire Position (1, 2, 3, 4).

● If we make each car/position combination a block, we have 16 blocks → we need 64 tires (inefficient and costly!)

● What if we only have 16 tires for the experiment?

A Poor Arrangement:

● Here, the value of car as a blocking factor is lost.

● Each car has only one brand of tire.

A Better Arrangement:

● **Now each car gets each brand of tire and each position gets each brand of tire.**

● **This design is called a Latin Square.**

● **Each row and each column contains each treatment once and only once.**

● **A $t \times t$ Latin Square is used for an experiment for t treatments and two blocking factors:**

- **Row factor with t levels**
- **Column factor with t levels**

Formal Linear Model for Latin Square:

<u>Note</u>: In a Latin Square design, there is assumed to be no interaction!

<u>Example</u> (Table 10.4): Experiment to study the effect of music type on employee productivity

- **Treatments: A = rock & roll, B = country, C = easy listening, D = classical, E = none.**

- **Row factor levels: 5 times of day (9-10, 10-11, 11-12, 1-2, 2-3)**

- **Column factor levels: 5 days of week (Mon, Tue, Wed, Thu, Fri)**

A 5 $\times$ 5 Latin Square is:

- **Each music type appears once on each day and once at each time of day.**

- **Testing for a significant effect of music type on mean productivity:**

F* =

- **There is a significant difference in mean productivity among the five music types.**

- **Note: There is also a significant row effect (time of day) and a significant column effect (day of week).**

● Specifically, which music types are significantly different?

● Using Tukey's procedure, we see:

<u>Summary</u>:

● Main advantage of a Latin Square design:
<u>Efficiency</u> – can perform useful tests with relatively few experimental units.

● Main disadvantage: cannot test for any interaction.