Chapter 21: Randomized Complete Block Design (RCBD)

Blocking:

• Consider a situation in which we have one factor of interest.

• If the experimental units are not homogeneous, there may be a great deal of variation in the response values, even among those having the same level of the factor.

• This leads to a large "experimental error variance" (measured by σ^2).

Example (Chemistry experiment):
Response: reaction rate of chemical agent
Factor: Chemical agent (5 treatments)

• Only 5 chemical agents can be tested each day.
• A different technician conducts the experiment each day.
• Wide variability among technicians →

Solution: To account for variation due to technician, we treat technician as a "blocking factor".
• We separate the experimental units into groups called blocks and then assign each treatment

Example 2: Interested in the effect of three different teaching methods on student learning.

Response: Improvement on a test score
Treatments: 3 methods (lecture; discussion; student presentation)

• But students will be taught be different instructors – we're not interested in the instructor effect, but it adds variability.

Solution:

Example 3: Lab animals of the same species given different diets and weight gain from birth is measured.
Experimental Units: animals
Response: Weight gain
Treatments: diets

Blocks:

Goal: Blocks should be chosen so that units in the same block are _____ but units in different blocks are

_____.

• When we use blocks, it reduces the unexplained variation – the

• This means there is more precision in estimating

Model for RCBD

• Assume there are n_b blocks and r treatments.

Equation:

• If the treatments are fixed,

• Often the blocks are random (they are randomly selected from a large population). In this case,

• If the blocks are fixed, then

• With this model, we separate the total variation in Y into three sources:

• The fitted values for the RCBD analysis are

and the residuals are

ANOVA Table for RCBD

• Note that the experimental error variation is measured by the SS for block × treatment interaction. If this interaction is significant (we can check this using Tukey's additivity test), we could
(1)

or (2)

• In the case of (2), when blocks are random we could still test for treatment effects using (see p. 1063)

but we would not be able to test for block effects.

Example (Executive data):

• Executives quantified a risk premium using 3 methods (utility, worry, comparison)
Subjects: 15 executives
Response: Degree of Confidence in Risk Premium
Treatments: the 3 Methods (U, W, C)
Blocks: Age Group of Executives (here, 5 _____ blocks)

Randomization scheme:

ANOVA Table from SAS:

• We may test whether the mean confidence is equal for the three methods:

• If block effects are of interest, we could test for significant block effects by comparing

• SAS gives the P-value for this test.

Some Model Diagnostics

• We may check model assumptions using

Example (Executive data):

Further Analysis of Treatment Effects

• We may further analyze differences among the treatment means using contrast or multiple comparisons.

Example (Executive data):

Note: If the blocks are considered random, we still use:
to test for significant variation across blocks.

• In this case we are testing:

• We may wish to do inference about

• When possible, more precise information in a RCBD can be attained (even if there is block × treatment interaction) if we have replication within each block (i.e., each treatment is repeated $d \geq 2$ times within each block).

• This is called a generalized random block design.

• Each block contains *dr* units.

• The analysis is identical to the two-factor ANOVA.

• MSE can be calculated and serves as an estimate of

• The denominator MS for F-tests again depends on which factors (here, blocks and treatments) are fixed or random (Table 25.6, page 1053 is again useful).

Example (Task Completion):
Response: Time to Complete Task
Blocks: Gender (2 levels, considered _____)
Treatments: Distraction Levels (2 levels, considered _____)

Balanced Incomplete Block Design (BIBD)

• When our resources are too limited to use a RCBD, we may use a BIBD.

• This block design is called incomplete because not all the treatments appear in each block.

• It is called balanced because each treatment appears in the same block with every other treatment the same number of times.

Example: r = 4 treatments (A,B,C,D) and n_T = 12 exper. units

• Suppose we can divide the units into 6 blocks.

• If we divide the units into 4 blocks, then:

• In practice, once we pick a design, we would randomly arrange the treatments within each block.

Advantages of BIBD:
• Can use block design when block size is smaller than the number of treatments.
• Same precision in estimating each treatment effect
• Scheffe and Tukey procedures can be used when the design is balanced.

Disadvantages of BIBD:
• BIBDs don't exist for every combination of number of treatments, number of blocks, and block size.
[A list of BIBDs for selected combinations of r, n_b, and r_b (block size) is given in Table B.15 (p. 1345-1347).]
• Must assume no treatment × block interaction.
• Analysis is more complicated than for RCBD.

• With BIBDs, out F-tests require the Reduced vs. Full Model approach with indicators for blocks and treatments (see p. 1177-1179).
• Can do this correctly in SAS (with PROC MIXED) or in R (see examples).

<u>Example</u> (Table 28.2 data):

• We wish to compare mean consumer ratings (the response) across five formulations of a cereal (the treatments).
• Since there is probably variation across consumers, we use the <u>ten</u> consumers as the blocks.
• For quality purposes, each consumer can only rate <u>three</u> formulations (must use <u>incomplete</u> design).

<u>SAS example</u>: If blocks are considered a random sample from a population of consumers, we use a

Example:

• If blocks are considered fixed, we can include them in the MODEL statement and omit the RANDOM statement.

Latin Square Designs

• These are efficient designs for situations in which we have two blocking factors.

Example: (Drug study)

Subjects: Patients
Response: Cholesterol Reduction
Treatments: 4 different drugs
Blocking Factors: Age Group (4 levels),
Blood Pressure Status (Low, Medium, High, Extreme)

• For a RCBD, even if we only have 1 observation per cell, we'd need

• May be infeasible to obtain that many patients.

• A Latin Square Design is arranged so that one blocking factor is the row factor and the other blocking factor is the column factor.

In a Latin Square:

• In the example above, a Latin Square would require

Advantages:
(1)

(2)

Disadvantages:
(1)

(2)

(3)

<u>Randomization Scheme</u>: For your particular value of *r*:

(1)

(2)

(3)

Example (*r* = 4):

<u>Example</u>: Assessing effect of background music on bank tellers' productivity:

<u>Experimental Units</u>: Working Days for Bank Crew
<u>Response</u>: Productivity Rating
<u>Treatments</u>: Type of Music (A = Slow Vocal, B= Medium Vocal, C = Fast Vocal, D = Medium Instrumental, E= Fast Instrumental)
<u>Row Factor</u>: Week (1, 2, 3, 4, 5)
<u>Column Factor</u>: Weekday (M, Tu, W, Th, F)

• A Latin Square design can complete the experiment in

After Randomization, the Design is:

Model for Latin Square Design

ANOVA table for Latin Square Model

• Formulas for the sums of squares given on p. 1189.

• To test for significant differences in mean response among the treatments, we test:

by comparing

SAS example (music data): Using $\alpha = 0.01$,

• Specific treatment comparisons can be investigating using contrasts or multiple comparisons.

Music example:

• If desired, effects of blocking factors can be tested using appropriate F-statistics from the ANOVA table.

Model assumptions may be checked via:

(1)

(2)

(3)

• If the treatments are random or if either blocking factor has random levels, the usual adjustments are made to the model.

• Section 28.7 discusses Latin Square designs with replication – more than one observation per cell.

• In some cases, multiple experimental units can be given the same treatment-row-column combination.
• In that case, we have replication within each cell, and we can formally test the fit of the additive (no-interaction) model using a lack-of-fit F-test (see example with Table 28.8 data).

• In other cases (such as the music/productivity experiment?), we cannot obtain multiple observations in each treatment-row-column combination.
• In that case, we can augment the experiment with more data by using multiple Latin Squares (each arrangement is selected independently).

<u>Note</u>: If the treatments in a Latin Square are factorial with two factors A and B, then the treatment SS can be decomposed as usual (SSTR = SSA + SSB + SSAB), to assess separately the effects of factors A and B and their interaction.