# Chapter 27: <u>Repeated Measures Designs</u>

**• Occur when several observations are taken (over time) <u>on the same subject</u>.**

**• For a group of *s* subjects, each subject is given a sequence of *r* treatments.**

**• Because observations on the same subject are likely to be similar, the subjects play the role of**

**<u>Example 1</u>: In several cities, a fast-food chain produces four different advertising campaigns (given in January, March, May, July). The sequencing of the campaigns is randomly chosen in each city.**

**<u>Response</u>: Sales for that month**
**<u>Subjects</u>:**

**<u>Example 2</u>: For a sample of sick patients, 5 different drugs are given in sequence over a period of time. The order of the drugs is randomly chosen for each patient.**

**<u>Response</u>: Improvement in white blood cell count**
**<u>Subjects</u>:**

**• The analysis is identical to that of a RCBD, with the subjects serving as blocks.**
**• Typically, the subjects are a random sample.**

**Model:**

**As with a mixed model,**

**• The ANOVA and tests are identical in this model to the RCBD analyses.**

**• If two observations near in time within a subject have a different correlation than two observations far apart in time within a subject, then this compound-symmetry assumption is <u>not</u> appropriate.**
**• More advanced methods must be used in that case (see the conservative test method given in Comment 2, pg. 1065).**

**• This compound-symmetry assumption can be examined by viewing the estimated within-subjects variance-covariance matrix, with entries:**

**Example (Wine data):**
**Checking model assumptions:**

**Inferences comparing the four wines:**

**• Section 27.3 discusses two-factor experiments with repeated measures on one of the factors.**
**Example (shoe data):   Response: Sales**
**Factor A: Type of Advertising Campaign**
**Factor B: Time (1 = before, 2 = during, 3 = after campaign)**
**Subjects: 10 test markets (chosen at random)**

**Note: Five of the test markets received campaign 1, and the other five received campaign 2 (subjects are "nested" within factor A – more about this later).**

**Note: If the data in such a study are unbalanced, the methods of Section 25.7 must be used (in SAS, use PROC MIXED in unbalanced case rather than PROC GLM).**

# Nonparametric Methods in ANOVA

• In ANOVA, sometimes the normality assumption for the response may not be reasonable (even after transformation?)
• Some rank-based distribution-free alternatives to the common ANOVA tests have been developed.

## Kruskal-Wallis Test

• An alternative to the one-way ANOVA F-test:

**Model:**

• We assume the $r$ populations are continuous and identical (in shape, variance, etc.) except possibly for their centers.

• Procedure: rank the entire data set from 1 (smallest) to $n_T$ (largest), in ascending order of response values. (If there are tied values, midranks are used.)
• Replace the response values with their ranks and perform the ANOVA calculations on the ranks.

The Kruskal-Wallis test statistic is

**• Our hypotheses are:**

**• For large samples (rule of thumb: )**

**Note: If the ranks inside one treatment vary greatly from the ranks inside other treatment(s):**

**• With small samples, tables/software are available for performing the K-W test based on the exact null distribution of $\chi^{2*}_{KW}$.**

**<u>Example</u> (Soil data):**

**<u>Response</u>: Percentage of clay in soil**
**<u>Factor</u>: Location (4 different levels)**
**• Six observations were made in each location.**

**Boxplots show**

**SAS/R Results:**

**• Bonferroni procedure provides simultaneous rank-based testing limits for**

**• For our example:**

# Friedman Test

• A distribution-free test for treatment effects for a RCBD.

**Model:**

• We assume each treatment appears <u>once</u> within each block.

• Block effects could be random; in that case, $\rho_i$ and $\varepsilon_{ij}$ need <u>not</u> have a normal distribution, merely a continuous distribution.

**Hypotheses:**

<u>Procedure</u>: Rank all responses <u>within each block</u> in ascending order, from 1 (smallest) to *r* (largest).

• Perform ANOVA calculations for RCBD on the <u>within-block ranks</u>.

**The Friedman test statistic is:**

• **For large samples (rule of thumb: )**

• **For small samples, tables of critical values are available for exact tests based on $\chi^{2*}_F$.**

**<u>Example</u> (Wind speed data):**

**<u>Response</u>: average wind speed reduction**
**<u>Treatments</u>: 5 different distances to shelterbelt (line of trees)**
**<u>Blocks</u>: 9 different months**

• **Is there a significant effect on mean wind speed reduction?**

**<u>Note</u>: When each treatment appears $d \geq 2$ times within each block, the <u>Mack-Skillings test</u> is an appropriate extension of Friedman's test.**

**<u>Note</u>: Cochran's test is a version of Friedman's test for binary responses.**

**<u>Note</u>: When $r$ = 2, the K-W test reduces to the _____________**

**When $r$ = 2, the Friedman test reduces to the _______________**