2.6 Tools for Counting Sample Points

- For more complicated cases, some results from combinatorial analysis are useful for counting sample points.

Result: Consider selecting an element from a set having m elements and selecting another from a set having n elements. Then there are ______ possible pairs that could be selected.

Note: This rule extends to any number of sets. For example: Consider selecting one of $\{a_1, \dots, a_m\}$, one of $\{b_1, \dots, b_n\}$, and one of $\{c_1, \dots, c_p\}$. Then there are ______ possible triplets that could be selected.

Example 1 (previous section): Tossing 3 coins.

Example 2: We roll a 6-sided die, a 4-sided die, and a 20-sided die.

Defn. (Permutation): A permutation is an ordered arrangement of a specific number of distinct objects. (Order matters!)

- We denote the number of possible permutations of n distinct objects, taken r at a time, as:

A Formula for P_r^n: How many ways can we fill r positions with n distinct objects?

Example 1: There are 24 members of a student club. Four members will be randomly chosen to be president, VP, secretary, and treasurer. How many different arrangements of officers are possible?

Example 2: The club needs a speaker at each of 3 meetings. Five possible speakers are available. How many speaker schedules are possible?

When Objects are Not All Distinct

Example 3: How many permutations of the letters of the name "BOBBY" are there?

Theorem: The number of permutations of n objects, in which n_1 are in the first group, n_2 in the second group, ..., n_k in the k-th group, and $n_1 + n_2 + \cdots + n_k = n$, is:

Example 3 again:

Note: Terms such as $\binom{n}{n_1 \cdots n_k}$ are called

______________ ______________.

Example 4: Consider a sequence of 10 coins, of which 2 are pennies, 5 are nickels, and 3 are dimes. How many distinguishable arrangements are possible?

Note: $\binom{n}{n_1\, n_2 \cdots n_k}$ also represents the number of ways of partitioning n objects into k groups containing $n_1, n_2, \dots, n_k$ objects, where each object is in exactly one group and $\sum_{i=1}^{k} n_i = n$.

Defn. Different selections of objects in which the order of objects does not matter are called combinations.

Theorem: The number of unordered subsets of size r chosen (without replacement) from n available objects is:

Proof:

Note: Terms such as $\binom{n}{r}$ are called

__________ ______________.

Example 5: There are 13 tenure-track statistics faculty. How many ways are there to select a 3-person committee?

2.7 Conditional Probability and Independent Events

- Consider randomly selecting a 40-year-old.
- Define event A = "the person will contract lung cancer in the next two decades".
- We could conceive of $P(A)$.
- Now consider event B = "the person is a smoker".
- Suppose we know event B is true: What is the probability of A, <u>given</u> B?
- This is a <u>conditional</u> probability, which we denote $P(A \mid B)$.
- It may be different from the <u>unconditional</u> probability $P(A)$.

<u>Defn:</u> $P(A \mid B) =$

Example 1: We roll a fair die:

Define A = "Roll a 5" and
B = "Roll an odd number".

Clearly $P(A) =$ and $P(B) =$

Given that B is true, what is the probability of A?

Note: For an event B with $P(B) > 0$, the probability set function $P(\cdot \mid B)$ satisfies the three Kolmogorov axioms. So all the probability laws we have derived also apply to the conditional probability $P(\cdot \mid B)$.

For example, $P(\overline{A} \mid B) =$

Defn. Two events A and B are independent if (and only if) any one of these is true:

(i)

(ii)

(iii)

Note: If any of (i), (ii) or (iii) holds, then the other equalities also hold.

- If any of these is false, then A and B are called dependent events.

Example 1 again:

Define C = "Roll a number greater than 4".
Are A and C independent?

Are B and C independent?

Note: If A and B are independent, then so are:
(a) $\bar{A}$ and B
(b) A and $\bar{B}$
(c) $\bar{A}$ and $\bar{B}$.

Proof of (a):

Defn: (Multiplicative Law of Probability)
The probability of the intersection of events A and B is

$$\begin{aligned} P(A \cap B) &= P(A)\,P(B \mid A) \\ &= P(B)\,P(A \mid B) \end{aligned}$$

Proof:

Corollary: If A and B are independent events, then $P(A \cap B) =$

Example 2: Suppose a basketball player makes 70% of her initial free throws. On her second attempt, she has an 80% success rate if she made the first and a 60% success rate if she missed the first. What is the probability she makes both of a pair of free throws?

Example 2(a): What is the probability she misses both free throws?