

2.9 Using Event-Composition to Calculate Probability

- This is a method for calculating probabilities that does not require listing all sample points in S explicitly.

Steps: ① Define the experiment.

② Visualize the nature of the sample points (you can write down a few)

③ Identify the event of interest and express it as a composition of two or more events, using intersections, unions, and complements.

④ Use the laws of probability (additive, multiplicative, etc.) to find the probability of the composition.

Note: You must be sure that the event of interest is equal to the composition of events.

Example 1: Suppose a radar trying to detect aircraft has a 0.02 chance of failing to detect a plane that is in its area. What is the probability it will correctly detect exactly three aircraft before failing to detect one, if aircraft arrivals are independent events?

Example 2: Suppose a lie detector will give a false positive 10% of the time when the subject is truthful and a correct positive 95% of the time when the subject is lying. Suppose there are two subjects; one is truthful and one is lying. If the detector acts independently for the two subjects:

- What is the probability of a positive reading for both subjects?

- What is the probability the detector is completely correct?

- What is the probability the detector is completely wrong?

- What is the probability of at least one positive reading?

2.10 The Law of Total Probability and Bayes' Rule

Defn: For some positive integer k , the collection of sets $\{B_1, B_2, \dots, B_k\}$ forms a partition of S if:

(i)

(ii)

Note: For any subset A of S and any partition $\{B_1, B_2, \dots, B_k\}$ of S , we can decompose A as:

$$A =$$

Venn Diagram:

Theorem: (Law of Total Probability)

If $\{B_1, B_2, \dots, B_k\}$ is a partition of S such that $P(B_i) > 0$, $i=1, \dots, k$, then for any event A ,

$$P(A) = \sum_{i=1}^k P(A|B_i)P(B_i)$$

Proof:

Corollary: For any event A and any event B such that $0 < P(B) < 1$, we have

$$P(A) =$$

Proof: Note

Theorem (Bayes' Rule): If $\{B_1, \dots, B_k\}$ is a partition of S , with $P(B_i) > 0$, $i=1, \dots, k$, then for any $j=1, \dots, k$:

$$P(B_j | A) = \frac{P(A | B_j) P(B_j)}{\sum_{i=1}^k P(A | B_i) P(B_i)}$$

Proof:

Corollary: If $0 < P(B) < 1$, then

$$P(B | A) = \frac{P(A | B) P(B)}{P(A | B) P(B) + P(A | \bar{B}) P(\bar{B})}$$

Proof:

Note: Bayes' Rule is useful for "reversing" conditional probabilities.

Example 1: A firm rents cars from 3 agencies: 60% from agency 1, 30% from agency 2, and 10% from agency 3. Suppose 9% of cars from agency 1 need a tuneup, 20% from agency 2 need a tuneup, and 6% from agency 3 need a tuneup. If the rental car delivered to an employee needs a tuneup, what is the chance it came from agency 2?

Example 2: Suppose 10% of all cars emit excessive pollutants. Suppose with probability 0.99, a car emitting excessive pollutants will fail the state emissions test. Suppose a car not emitting excessively will fail the test with probability 0.05. If a car fails the test, what is the probability that it is actually emitting excessive pollutants?