3.8 The Poisson Distribution

- Consider a r.v. Y that is a count of the number of occurrences of some phenomenon during some fixed unit of time (or space).

Examples: Y = number of phone calls received per day

Y = number of spots per square inch of an orange

Y = number of accidents per week at an intersection

- If we divide this interval of time (or space) into tiny subintervals such that:

and occurrences are independent across subintervals, then we have a

____________ experiment.

- The count of occurrences Y is the count of subintervals containing an occurrence.
- Suppose we have n such subintervals.
- Let the mean number of occurrences in the <u>full</u> <u>interval</u> be fixed at

and let the number of subintervals $n \to \infty$ (meaning that).

Then $p(y) =$

Defn. A r.v. Y has a Poisson distribution [Shorthand: $Y \sim \text{Pois}(\lambda)$] if its probability function is:

Theorem: The Poisson distribution is a valid probability distribution.

Proof:

Theorem: If $Y \sim \text{Pois}(\lambda)$, then:

Proof:

- The variance result will be proved later.

Example 1: Suppose the number of accidents per month at an industrial plant has a Poisson distribution with mean 2.6. Find the probability that there will be 4 accidents in the next month.

Find the probability of two or more accidents in the next month.

- Tedious cumulative Poisson probabilities can be calculated using Table 3 in Appendix 3 or using R:

<u>Previous Example</u>: Find the probability of having between 3 and 6 accidents.

What is the probability of 10 accidents in the next half-year?

- Note that

- <u>General Rule</u>: If the count of the number of occurrences in a unit of time follows a Poisson (θ) distribution, then the count of the number of occurrences in t units of time follows a

Relationship with Binomial Distribution

- If n is large, p is small, and $\lambda = np$ is somewhat small (book:), then the Bin(n, p) probabilities are approximately equal to the Pois(λ) probabilities.

Example: Suppose 3% of all USC students are vegetarians. Select 100 USC students at random. Find the probability of selecting at least five vegetarians.

Binomial:

Poisson: